THE UNIVERSITY OF TEXAS AT DALLAS

Generative Neural Networks

CS 6384 Computer Vision
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Slides borrowed from Professor Yu Xiang

Supervised Learning

$\mathbf{x}$ ⇒  ⇒ $\mathbf{y}$

Image

Object class

$$f(\mathbf{x})$$

Training Data $\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{N}$

Input

Output

Unsupervised Learning

Training data $\{\mathbf{x}_i\}_{i=1}^N$ No label

Goal: discover some underlying hidden structure of the data

Examples

- Dimension reduction
- Clustering
- Probability density estimation
- Generative models

Dimension Reduction

Map data from a high-dimension space to a low-dimension space

$$\mathbf{x} \in \mathcal{R}^n \rightarrow \mathbf{y} \in \mathcal{R}^m \qquad m < n$$

The low-dimensional representation maintains meaningful properties of the original data

- E.g., can be used to reconstruct the original data

Applications

- Data compression, data visualization, data representation learning

Principal Component Analysis (PCA)

Linear mapping

$$\mathbf{y} = P\mathbf{x}$$

$$m \times 1 \quad m \times m \quad m \times 1$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_m \end{bmatrix} \mathbf{x}$$

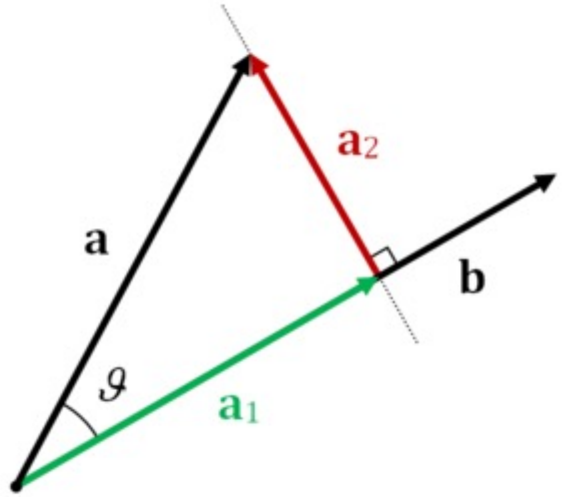
Rows of P, principal components

Principal Component Analysis (PCA)

Change of basis

$$\mathbf{y} = \begin{bmatrix} \mathbf{p}_1 \cdot \mathbf{x} \\ \mathbf{p}_2 \cdot \mathbf{x} \\ \vdots \\ \mathbf{p}_m \cdot \mathbf{x} \end{bmatrix}$$

Projection



$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos\theta$$

$$\mathbf{a}_1 = \|\mathbf{a}\| \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|}$$

If $\|\mathbf{b}\| = 1$ $\quad \mathbf{a}_1 = \mathbf{a} \cdot \mathbf{b}$

Principal Component Analysis (PCA)

Given a set of data points $X \in \mathcal{R}^{m \times n}$

dimension # data points

$$Y = PX$$

Covariance matrix

$$\mathbf{X} = \begin{bmatrix} \mathbf{x_1} \\ \vdots \\ \mathbf{x_m} \end{bmatrix} \qquad \mathbf{C_X} \equiv \frac{1}{n}\mathbf{X}\mathbf{X}^T \qquad \mathbf{C_Y}$$

Rows of X

Principal Component Analysis (PCA)

D is a diagonal matrix
E is a matrix of eigenvectors of C_X arranged as rows

The goal of PCA

- All off-diagonal terms in $\mathbf{C_Y}$ should be zero (Y is decorrelated)
- Each successive dimension of Y should be rank-ordered according to variance

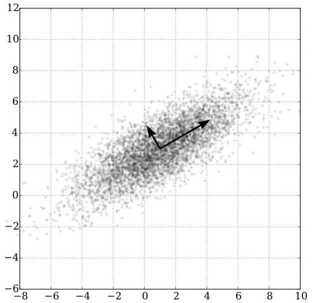
Solution

$$
\begin{aligned}
\mathbf{C_Y} &= \frac{1}{n}\mathbf{Y}\mathbf{Y}^T \\
&= \frac{1}{n}(\mathbf{PX})(\mathbf{PX})^T \\
&= \frac{1}{n}\mathbf{P}\mathbf{X}\mathbf{X}^T\mathbf{P}^T \\
&= \mathbf{P}(\frac{1}{n}\mathbf{X}\mathbf{X}^T)\mathbf{P}^T \\
\mathbf{C_Y} &= \mathbf{P}\mathbf{C_X}\mathbf{P}^T
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{C_Y} &= \mathbf{P}\mathbf{C_X}\mathbf{P}^T \\
&= \mathbf{P}(\mathbf{E}^T\mathbf{D}\mathbf{E})\mathbf{P}^T \\
&= \mathbf{P}(\mathbf{P}^T\mathbf{D}\mathbf{P})\mathbf{P}^T \\
&= (\mathbf{P}\mathbf{P}^T)\mathbf{D}(\mathbf{P}\mathbf{P}^T) \\
&= (\mathbf{P}\mathbf{P}^{-1})\mathbf{D}(\mathbf{P}\mathbf{P}^{-1}) \\
\mathbf{C_Y} &= \mathbf{D}
\end{aligned}
$$

The principal components P is the eigenvectors of

$$\mathbf{C_X} \equiv \frac{1}{n}\mathbf{X}\mathbf{X}^T$$



Principal Component Analysis (PCA)

Dimension reduction

$$\mathbf{y} = P\mathbf{x}$$

$m \times 1$ $\quad m \times m$ $\quad m \times 1$

$$\mathbf{y} = P_L\mathbf{x}$$

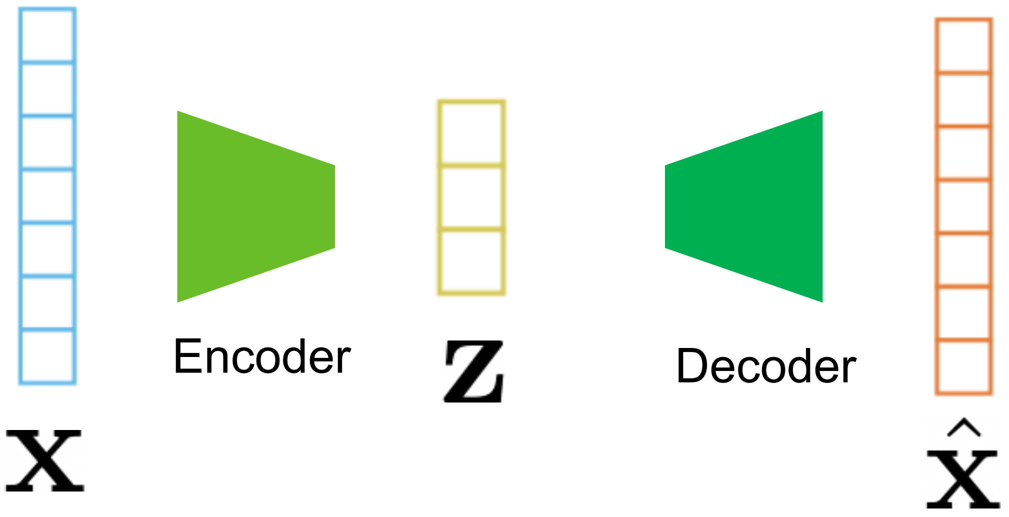
$$\mathbf{y} = \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_L \end{bmatrix} \mathbf{x}$$

$L \times 1$

Use L < m principal components

Autoencoder

Use a neural network for dimension reduction

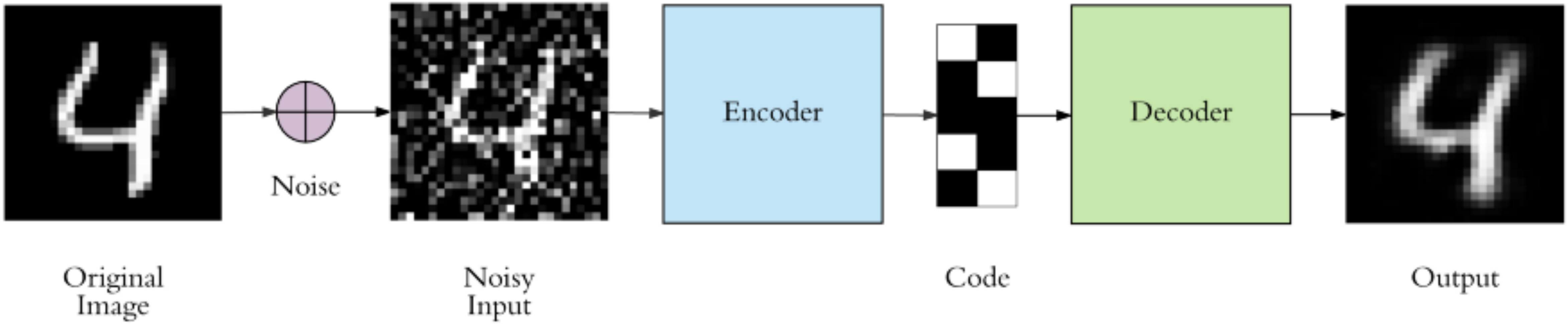


$$\mathbf{z} = f(\mathbf{x}) \qquad \hat{\mathbf{x}} = g(\mathbf{z})$$

Reconstruction loss function

$$L_2 = ||\mathbf{x} - \hat{\mathbf{x}}||^2$$

Case Study: Denoising Autoencoder



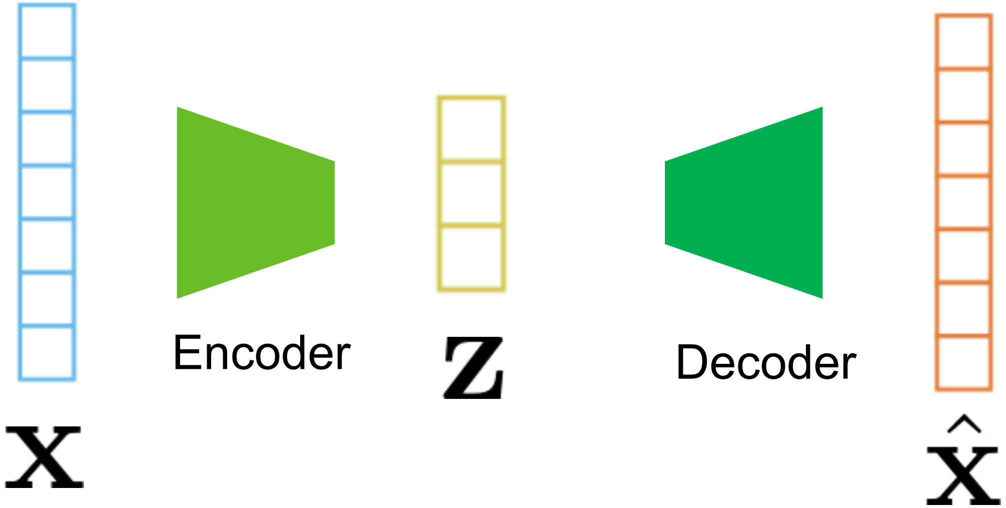
https://www.analyticsvidhya.com/blog/2021/07/image-denoising-using-autoencoders-a-beginners-guide-to-deep-learning-project/

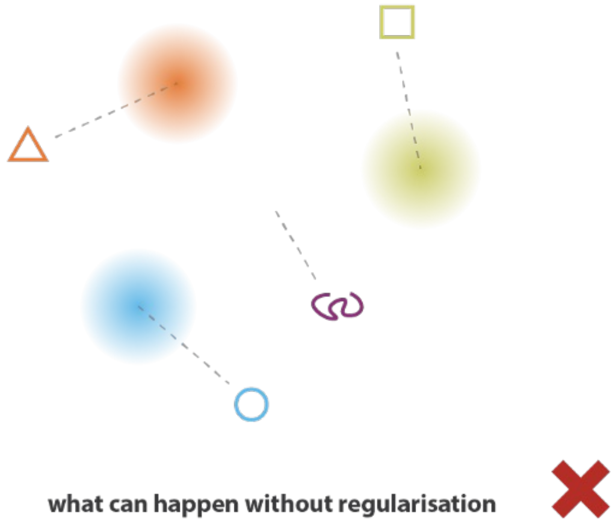
Content Generation

Given a dataset $\{\mathbf{x}_i\}_{i=1}^{N}$

How to generate new content from the underlying distribution P(x)?

Autoencoder is not suitable for content generation

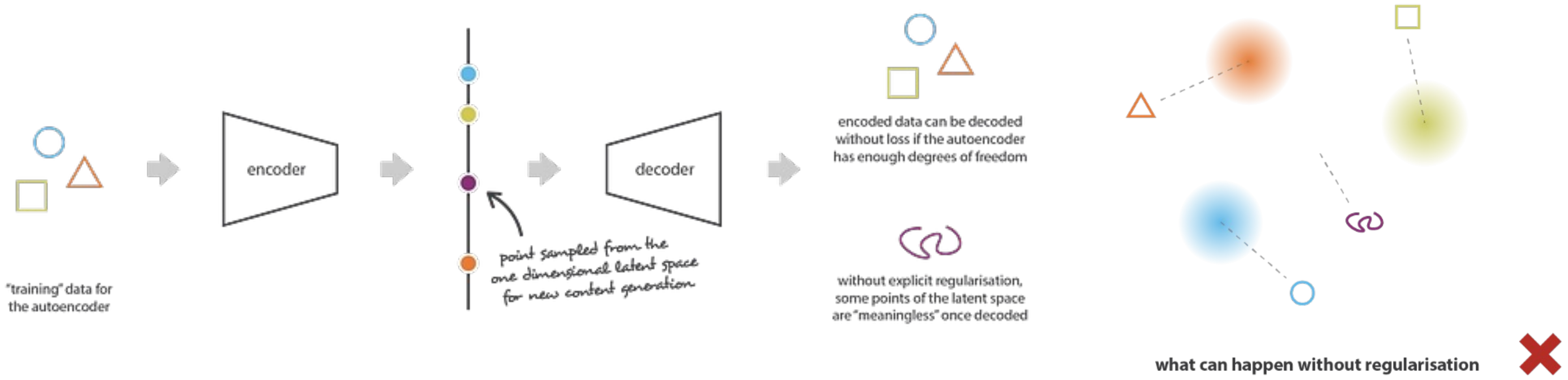




The latent space is not regularized. Some latent vectors may generate meaningless content.

Autoencoder is not suitable for content generation

- Irregular latent space prevent us from using autoencoder for new content generation

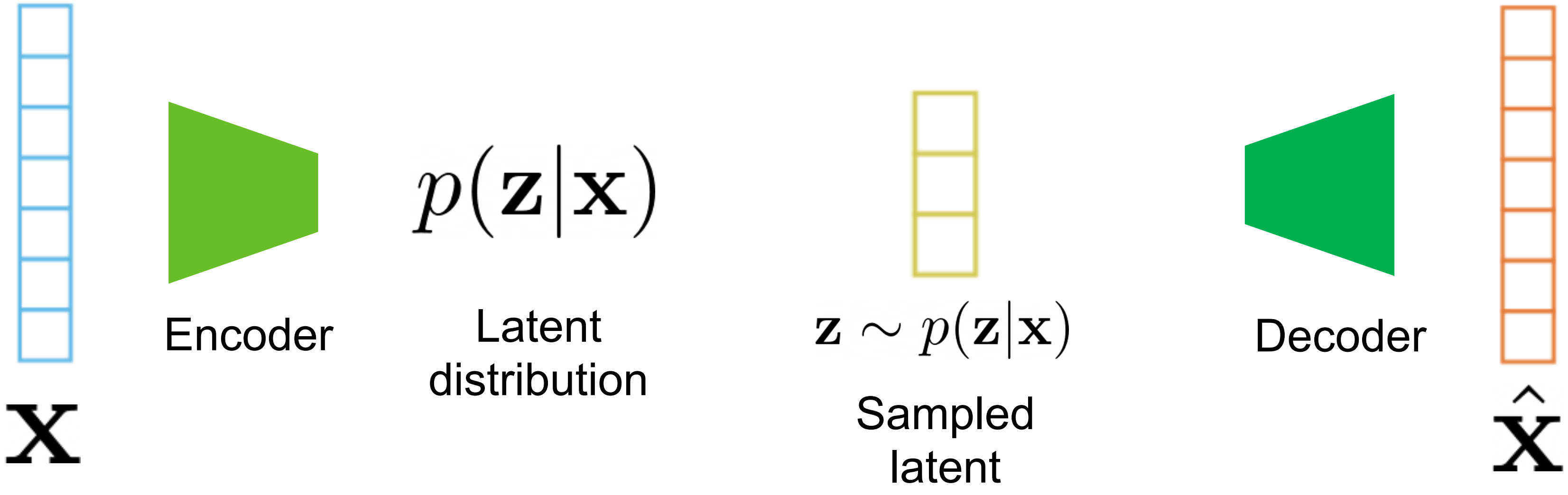


The latent space is not regularized. Some latent vectors may generate meaningless content.

Variational Autoencoder

Introduce regularization to the latent space

Probabilistic formulation

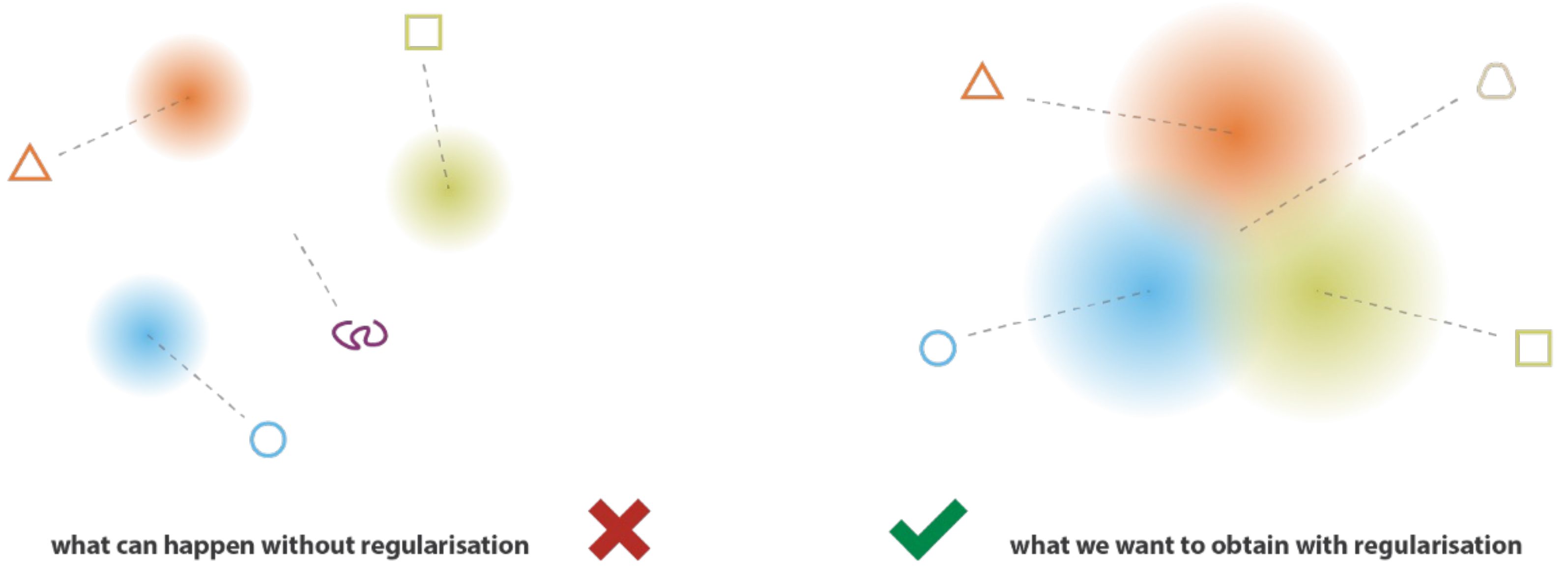


$$p(\mathbf{z}|\mathbf{x}) = \mathcal{N}(\mu_x, \sigma_x) \Longleftrightarrow \mathcal{N}(\mathbf{0}, \mathbf{I})$$ Prior distribution

Variational Autoencoder

Latent space

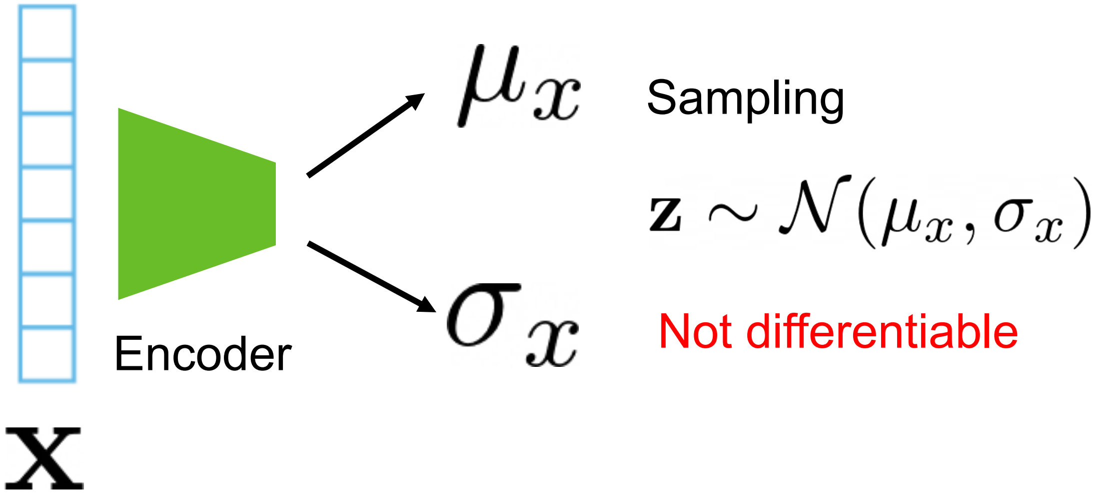
- Continuity (close points in latent space decode similar outputs)
- Completeness (a sampled latent should generate meaningful output)



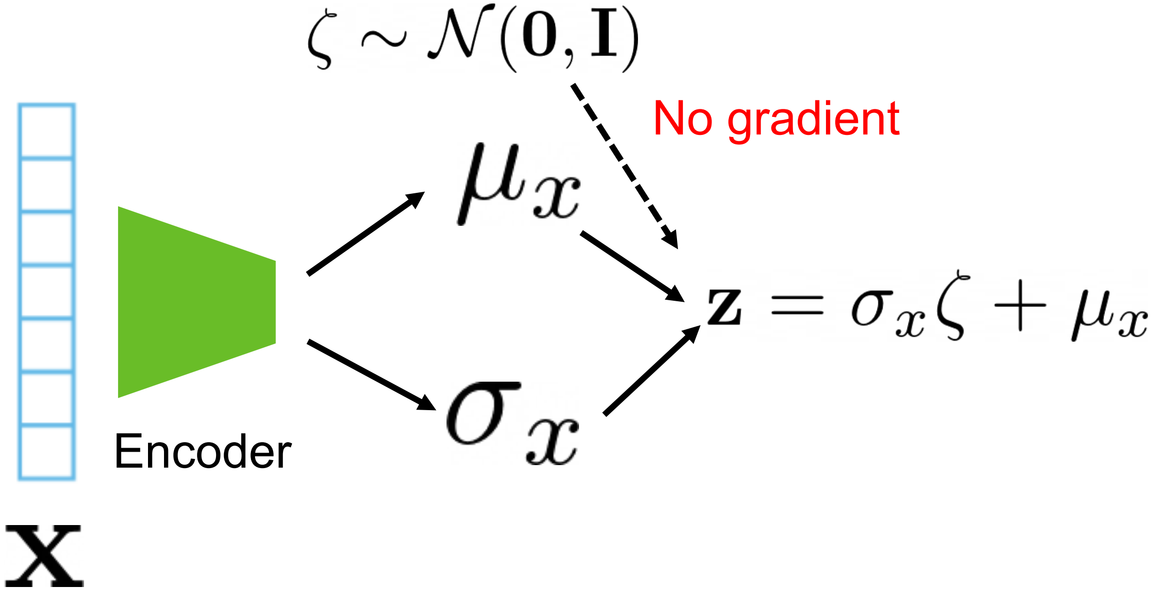
https://towardsdatascience.com/understanding-variational-autoencoders-vaes-f70510919f73

Variational Autoencoder

Encoder

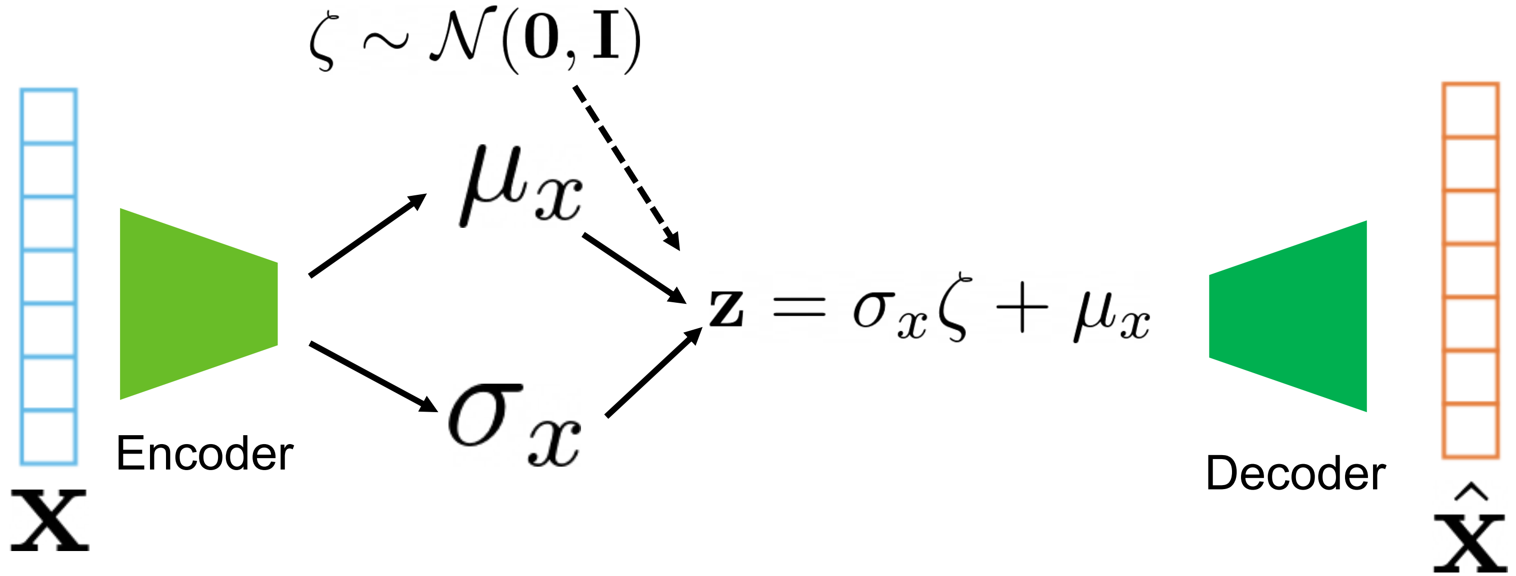


Reparameterization



Variational Autoencoder

Encoder-Decoder



Loss function

$$L = C\|\mathbf{x} - \hat{\mathbf{x}}\|^2 + \mathrm{KL}(\mathcal{N}(\mu_x, \sigma_x), \mathcal{N}(\mathbf{0}, \mathbf{I}))$$

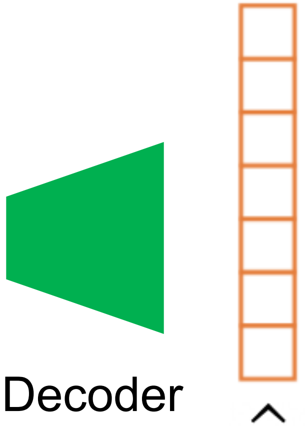
Reconstruction loss

Prior loss $D_{\mathrm{KL}}(P \parallel Q) = \int_{-\infty}^{\infty} p(x) \log\left(\frac{p(x)}{q(x)}\right) dx$

Variational Autoencoder

Generating data

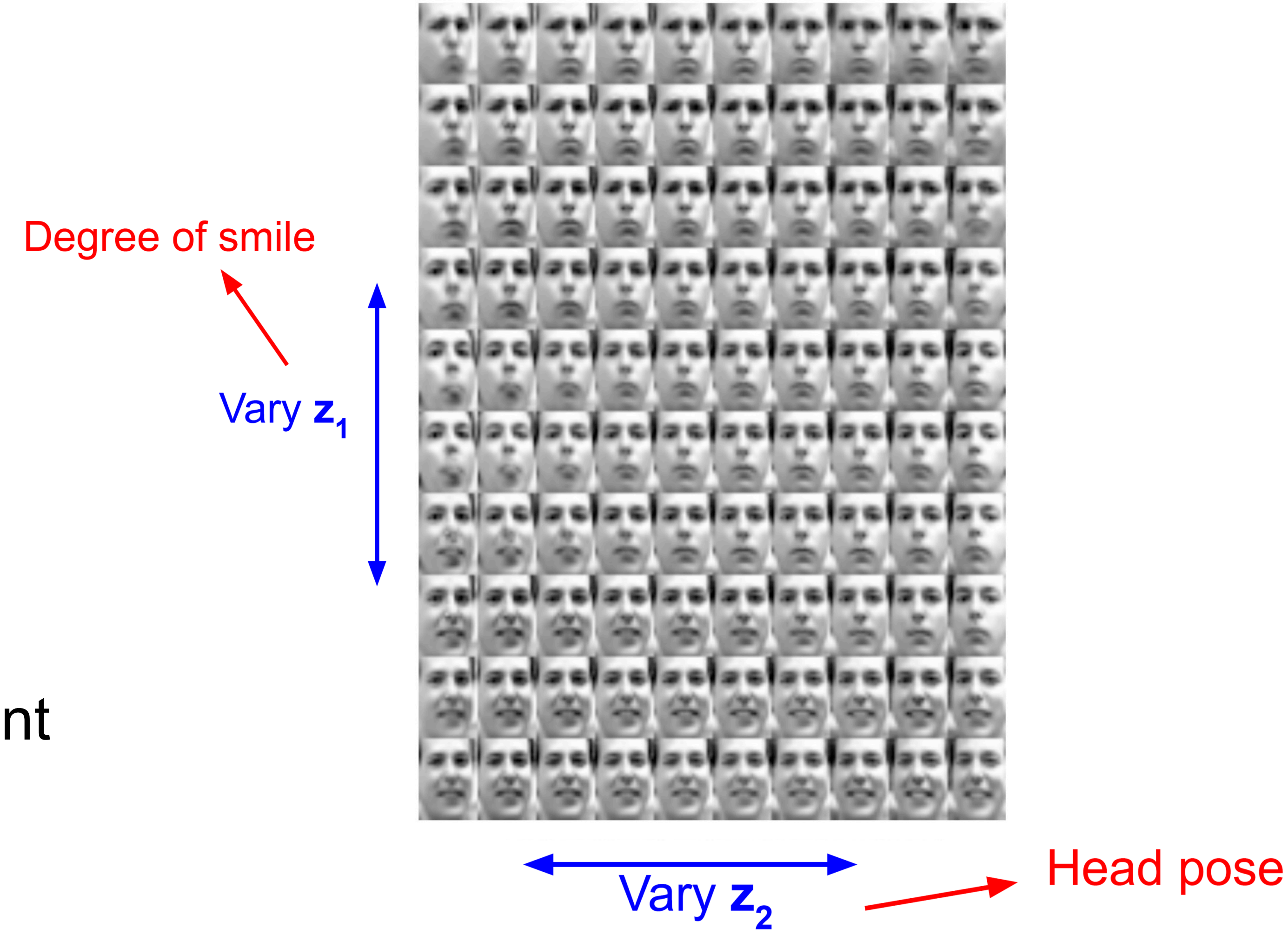
$$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$



$\hat{\mathbf{x}}$

- Diagonal prior on z -> independent latent variables
- Different dimensions of z encode interpretable factors of variation

2D latent space



Auto-Encoding Variational Bayes. Kingma & Welling, ICLR’14.

Direct Content Generation

VAE models the density as

$$p_\theta(x) = \int p_\theta(z) p_\theta(x|z) dz$$

Directly sample from the training distribution without modeling the probability density

Generative Adversarial Networks (GANs) can generate better samples compared to VAEs

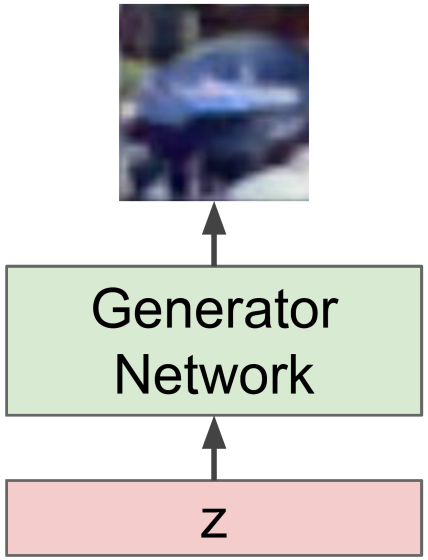
Generative Adversarial Network (GAN)

Goal: sample examples from training distribution $P(\mathbf{x})$

Solution

- First sample from a simple distribution (e.g., uniform distribution)
- Learn transformation to the training distribution

Output: sample from the training distribution



Input: random noise

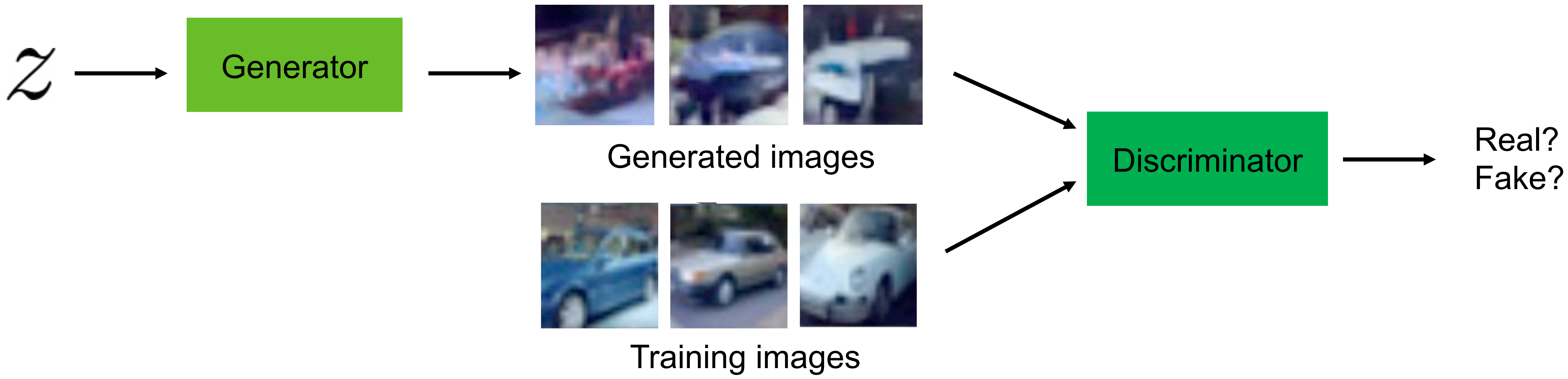
How to train the generator?

- We do not know the mapping from z to training data

Generative Adversarial Network (GAN)

Generator-Discriminator

Training GAN: Two-player Game



Discriminator: try to distinguish between real image and fake images (generated images from the generator)

Generator: try to fool the discriminator by generating real-look images

Training GAN: Two-player Game

Minmax objective function

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

Discriminator output for real data x

Discriminator output for generated fake data

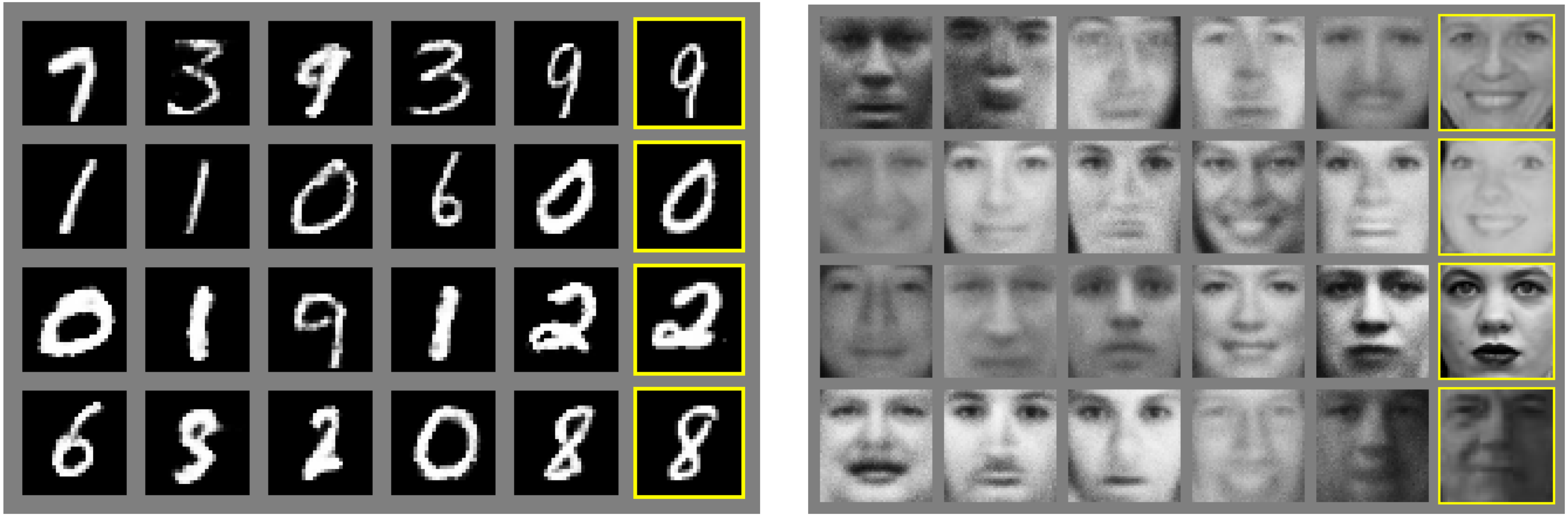
Generator output

- Discriminator: **maximize** the objective such that D(x) is close to 1 and D(G(z)) is close to 0
- Generator: **minimize** the objective such that D(G(z)) is close to 1 (fool the discriminator)

Generative Adversarial Nets. Goodfellow et al. NeurIPS'14

Generative Adversarial Network (GAN)

Visualization of samples from the model



Nearest neighbor from training set

Generative Adversarial Nets. Goodfellow et al. NeurIPS’14

Summary

Autoencoder

- Good for dimension reduction, cannot generate new data

Variational autoencoder

- Probabilistic formulation
- Regularized latent space, can be used to generate new data

Generative Adversarial Network

- Directly sample training distribution to generate data
- Better samples compared VAEs

Further Reading

A Tutorial on Principal Component Analysis. Jonathon Shlens, 2014.
https://arxiv.org/abs/1404.1100

Auto-Encoding Variational Bayes. Kingma & Welling, ICLR, 2004.
https://arxiv.org/abs/1312.6114

Autoencoders. Dor Bank, Noam Koenigstein, Raja Giryes, 2021.
https://arxiv.org/abs/2003.05991

Generative Adversarial Nets. Goodfellow et al. NeurIPS'14. https://arxiv.org/abs/1406.2661

UNSUPERVISED REPRESENTATION LEARNING WITH DEEP CONVOLUTIONAL GENERATIVE ADVERSARIAL NETWORKS. Radford et al., ICLR'16. https://arxiv.org/abs/1511.06434