THE UNIVERSITY OF TEXAS AT DALLAS

Structure from Motion and SLAM

CS 6384 Computer Vision

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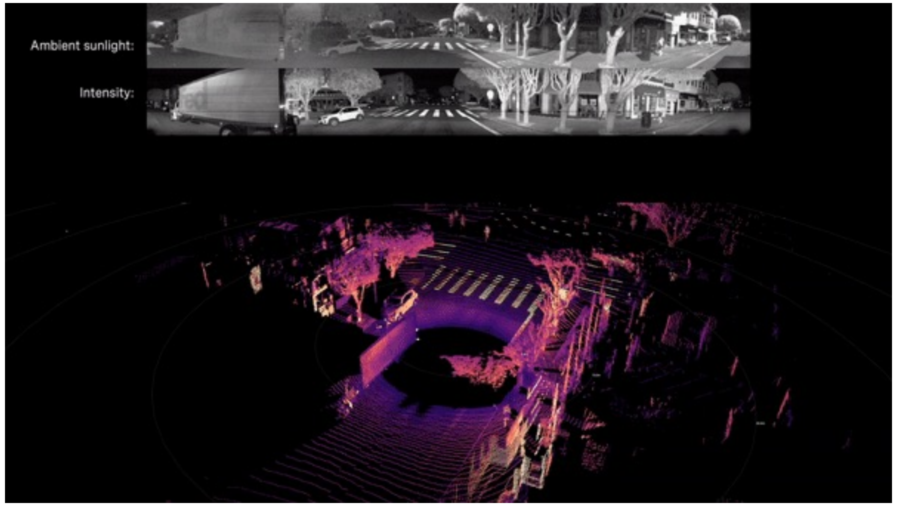
Slides borrowed from Professor Yu Xiang

How to Recover the 3D World from Images?

Structure from Motion (SfM)

- Structure: the geometry of the 3D world
- Motion: camera motion
- Input: a set of images (no need to be videos)
- From computer vision



Point cloud captured on an Ouster OS1-128 digital lidar sensor

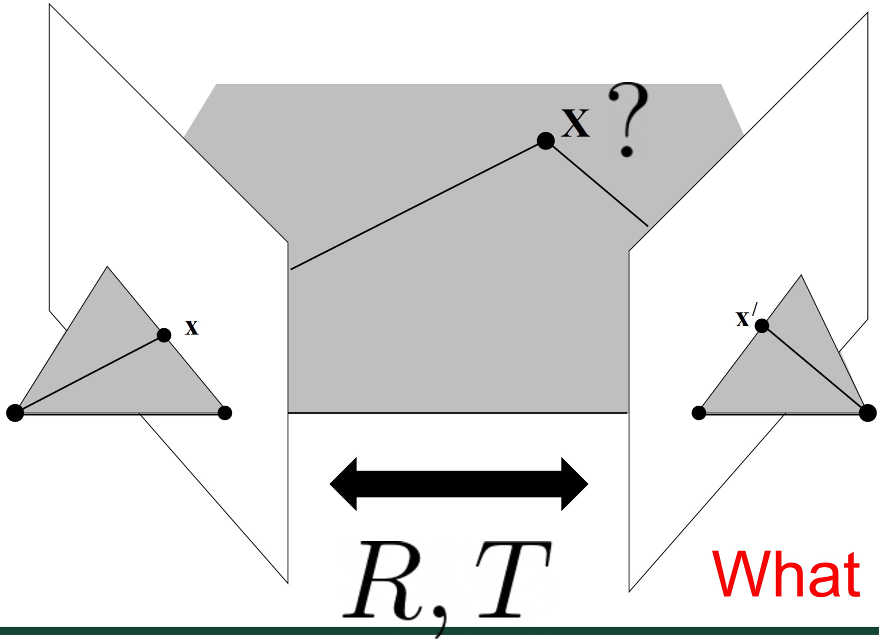
Simultaneous Localization and Mapping (SLAM)

- Localization: camera pose
- Mapping: build the geometry of the 3D world
- Input: video sequences
- From robotics

Triangulation

Idea: using images from different views and feature matching

Triangulation from pixel correspondences to compute 3D location



Given $\mathbf{x} \longleftrightarrow \mathbf{x}'$

Intersection of two backprojected lines

$$\mathbf{X} = \mathbf{l} \times \mathbf{l}'$$

What if unknow camera pose?

Colosseum in Rome

Structure from Motion

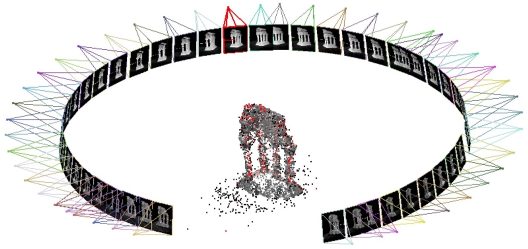
Input

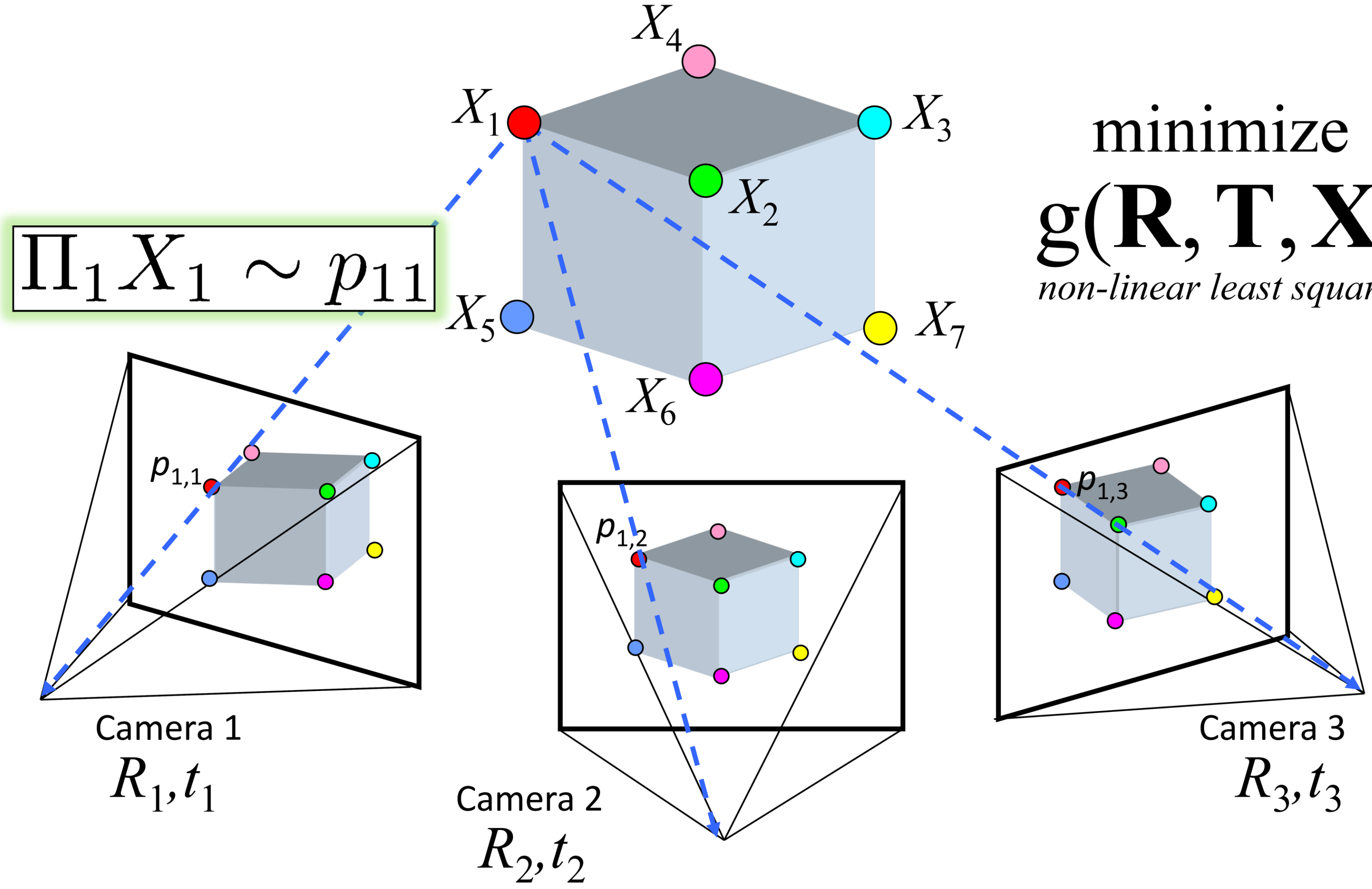
- A set of images from different views

Output

- 3D Locations of all feature points in a world frame
- Camera poses of the images

Structure from motion



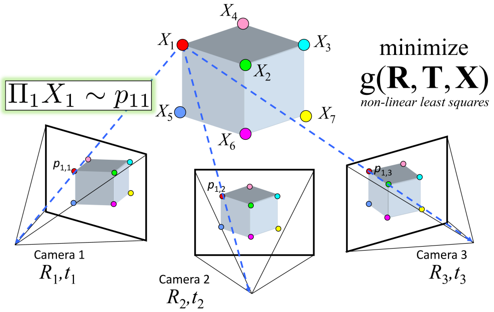


minimize

$$g(\mathbf{R}, \mathbf{T}, \mathbf{X})$$

non-linear least squares

Structure from Motion



Minimize sum of squared reprojection errors

$$g(\mathbf{X}, \mathbf{R}, \mathbf{T}) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} \cdot \left\| \underbrace{\mathbf{P}(\mathbf{x}_i, \mathbf{R}_j, \mathbf{t}_j)}_{\text{predicted image location}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\text{observed image location}} \right\|^2$$

m points, n images

Indicator variable:
is point i visible in image j?

Projection

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \mathbf{R}\mathbf{x} + \mathbf{t}$$

$$u' = f_x \frac{x'}{z'} + p_x$$

$$v' = f_y \frac{y'}{z'} + p_y$$

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \mathbf{P}(\mathbf{x}, \mathbf{R}, \mathbf{t})$$

Structure from Motion

How to minimize

$$g(\mathbf{X}, \mathbf{R}, \mathbf{T}) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} \cdot \left\| \mathbf{P}(\mathbf{x}_i, \mathbf{R}_j, \mathbf{t}_j) - \begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix} \right\|^2$$

A non-linear least squares problem (why?)

- E.g. Levenberg-Marquardt

The Levenberg-Marquardt Algorithm

Nonlinear least squares $\hat{\boldsymbol{\beta}} \in \operatorname{argmin}_{\boldsymbol{\beta}} S(\boldsymbol{\beta}) \equiv \operatorname{argmin}_{\boldsymbol{\beta}} \sum_{i=1}^{m} \left[y_i - f(x_i, \boldsymbol{\beta})\right]^2$

An iterative algorithm

- Start with an initial guess β_0
- For each iteration $\beta \leftarrow \beta + \delta$

How to get δ ?

- Linear approximation $f(x_i, \boldsymbol{\beta} + \boldsymbol{\delta}) \approx f(x_i, \boldsymbol{\beta}) + \mathbf{J}_i \boldsymbol{\delta}$ $\quad \mathbf{J}_i = \frac{\partial f(x_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}}$
- Find to δ minimize the objective $S(\boldsymbol{\beta} + \boldsymbol{\delta}) \approx \sum_{i=1}^{m} \left[y_i - f(x_i, \boldsymbol{\beta}) - \mathbf{J}_i \boldsymbol{\delta}\right]^2$

Wikipedia

The Levenberg-Marquardt Algorithm

Vector notation for $S(\boldsymbol\beta + \boldsymbol\delta) \approx \sum_{i=1}^{m} \left[y_i - f(x_i, \boldsymbol\beta) - \mathbf{J}_i \boldsymbol\delta\right]^2$

$$\begin{aligned} S(\boldsymbol\beta + \boldsymbol\delta) &\approx \|\mathbf{y} - \mathbf{f}(\boldsymbol\beta) - \mathbf{J}\boldsymbol\delta\|^2 \\ &= \left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta) - \mathbf{J}\boldsymbol\delta\right]^{\mathrm{T}} \left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta) - \mathbf{J}\boldsymbol\delta\right] \\ &= \left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right]^{\mathrm{T}} \left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right] - \left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right]^{\mathrm{T}} \mathbf{J}\boldsymbol\delta - (\mathbf{J}\boldsymbol\delta)^{\mathrm{T}} \left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right] + \boldsymbol\delta^{\mathrm{T}}\mathbf{J}^{\mathrm{T}}\mathbf{J}\boldsymbol\delta \\ &= \left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right]^{\mathrm{T}} \left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right] - 2\left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right]^{\mathrm{T}} \mathbf{J}\boldsymbol\delta + \boldsymbol\delta^{\mathrm{T}}\mathbf{J}^{\mathrm{T}}\mathbf{J}\boldsymbol\delta. \end{aligned}$$

https://www.cs.ubc.ca/~schmidtm/Courses/340-F16/linearQuadraticGradients.pdf

Take derivation with respect to δ and set to zero $\left(\mathbf{J}^{\mathrm{T}}\mathbf{J}\right)\boldsymbol\delta = \mathbf{J}^{\mathrm{T}}\left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right]$

Levenberg's contribution $\left(\mathbf{J}^{\mathrm{T}}\mathbf{J} + \lambda\mathbf{I}\right)\boldsymbol\delta = \mathbf{J}^{\mathrm{T}}\left[\mathbf{y} - \mathbf{f}(\boldsymbol\beta)\right]$ damped version

$$\beta \leftarrow \beta + \delta$$

Wikipedia

Structure from Motion

$$g(\mathbf{X},\mathbf{R},\mathbf{T}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \underbrace{w_{ij}}_{} \cdot \left\| \underbrace{\mathbf{P}(\mathbf{x}_i,\mathbf{R}_j,\mathbf{t}_j)}_{\substack{\textit{predicted} \\ \text{image location}}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\substack{\textit{observed} \\ \text{image location}}} \right\|^2$$

indicator variable:
is point *i* visible in image *j* ?

$$\beta = (\mathbf{X},\mathbf{R},\mathbf{T})$$

How to get the initial estimation β_0 ?

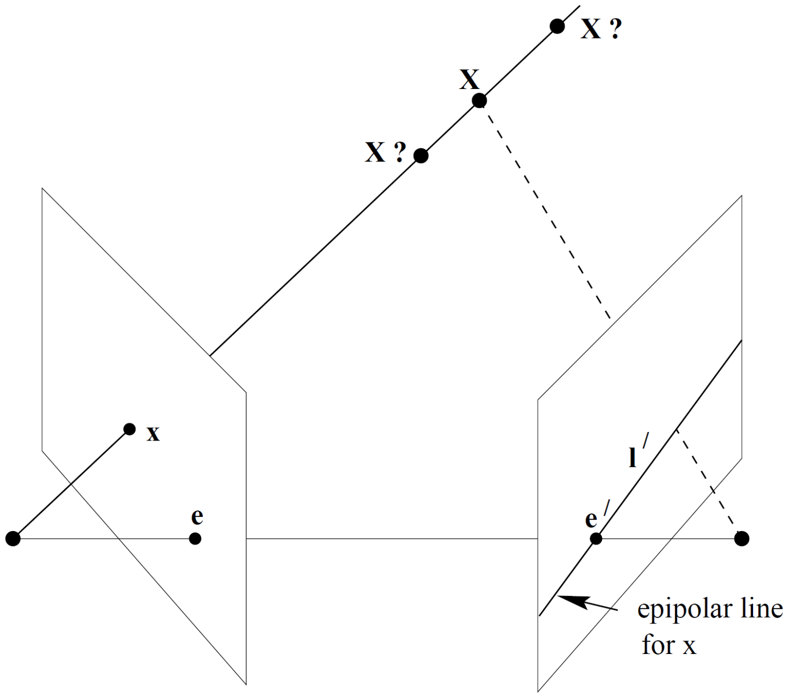
Random guess is not a good idea.

Matching Two Views

Fundamental matrix

$\mathbf{x}'$ is on the epiploar line $\mathbf{l}' = F\mathbf{x}$

$$\mathbf{x}'^T F \mathbf{x} = 0$$



$$\begin{bmatrix} x_i' & y_i' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = 0$$

$$x_i x_i' f_{11} + x_i y_i' f_{21} + x_i f_{31} + y_i x_i' f_{12} + y_i y_i' f_{22} + y_i f_{32} + x_i' f_{13} + y_i' f_{23} + f_{33} = 0$$

$$\begin{bmatrix} x_1 x_1' & x_1 y_1' & x_1 & y_1 x_1' & y_1 y_1' & y_1 & x_1' & y_1' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_m x_m' & x_m y_m' & x_m & y_m x_m' & y_m y_m' & y_m & x_m' & y_m' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{21} \\ f_{31} \\ f_{12} \\ f_{22} \\ f_{32} \\ f_{13} \\ f_{23} \\ f_{33} \end{bmatrix} = 0$$

We need 8 points to solve this system.

Matching Two Views

$$\mathbf{x}'^T F \mathbf{x} = 0$$

If we know camera intrinsics in SfM

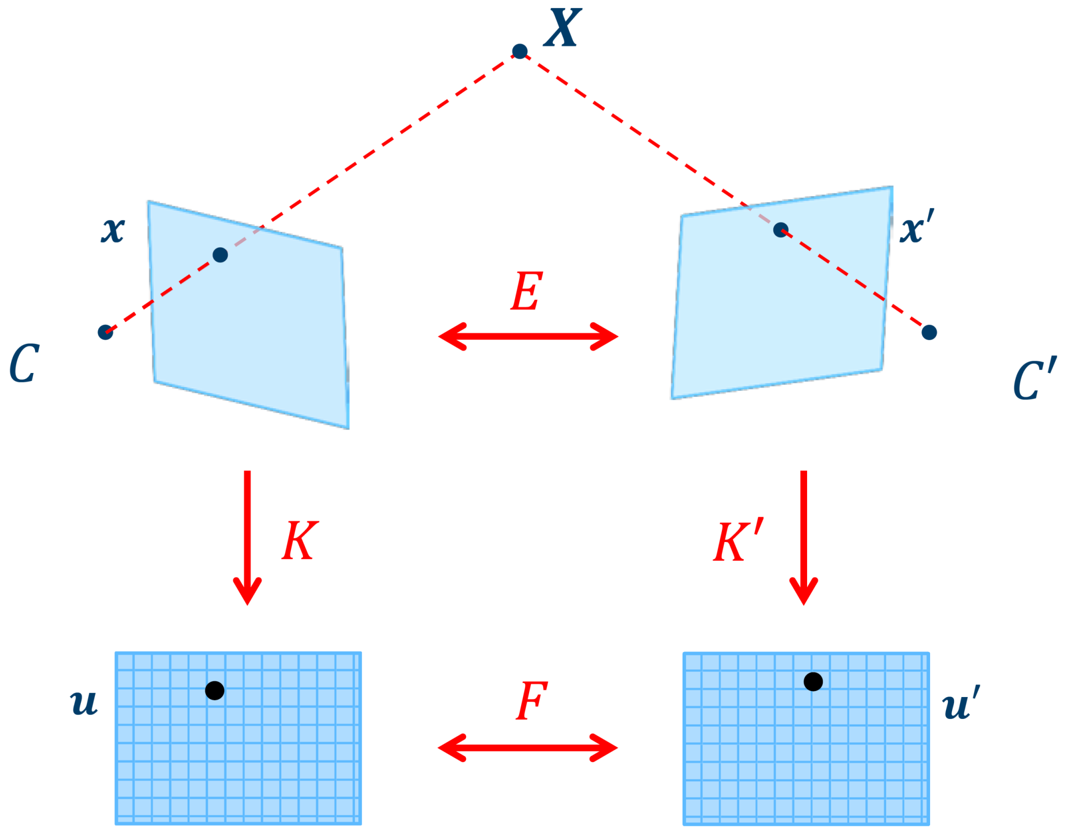
$$(K'^{-1}\mathbf{x}')^T E (K^{-1}\mathbf{x}) = 0$$

Normalized coordinates

$$F = K'^{-T} E K^{-1}$$

Essential matrix E

$$E = K'^T F K$$



Credit: Thomas Opsahl

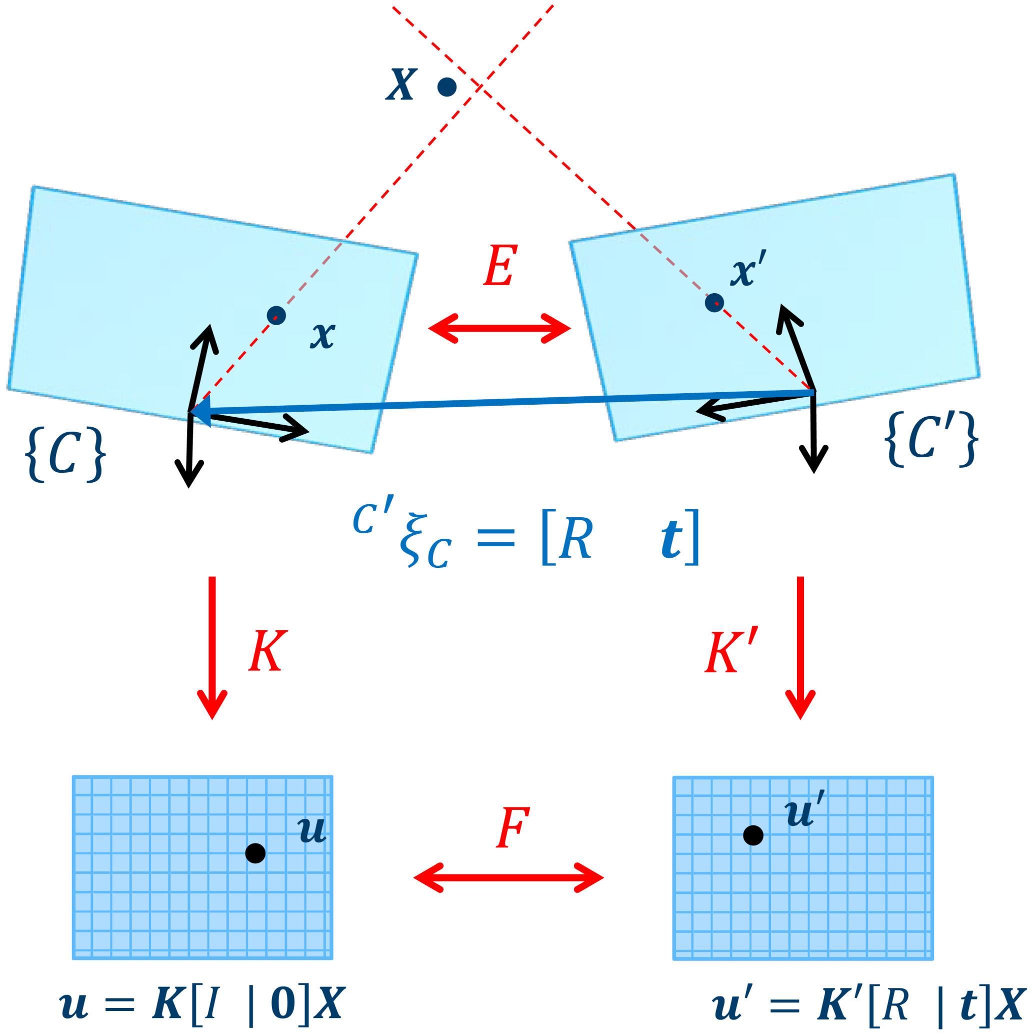
Matching Two Views

Recover the relative pose R and $\boldsymbol{t}$ from the essential matrix E up to the scale of $\boldsymbol{t}$

$$\mathrm{F} = [\mathrm{e}']_\times \mathrm{K}'\mathrm{R}\mathrm{K}^{-1} = \mathrm{K}'^{-\mathsf{T}}[\mathrm{t}]_\times \mathrm{R}\mathrm{K}^{-1}$$

$$E = K'^T F K$$

$$\mathrm{E} = [\mathrm{t}]_\times \mathrm{R}$$



Credit: Thomas Opsahl

H. C Longuet-Higgins, *A computer algorithm for reconstructing a scene from two projections*, 1981

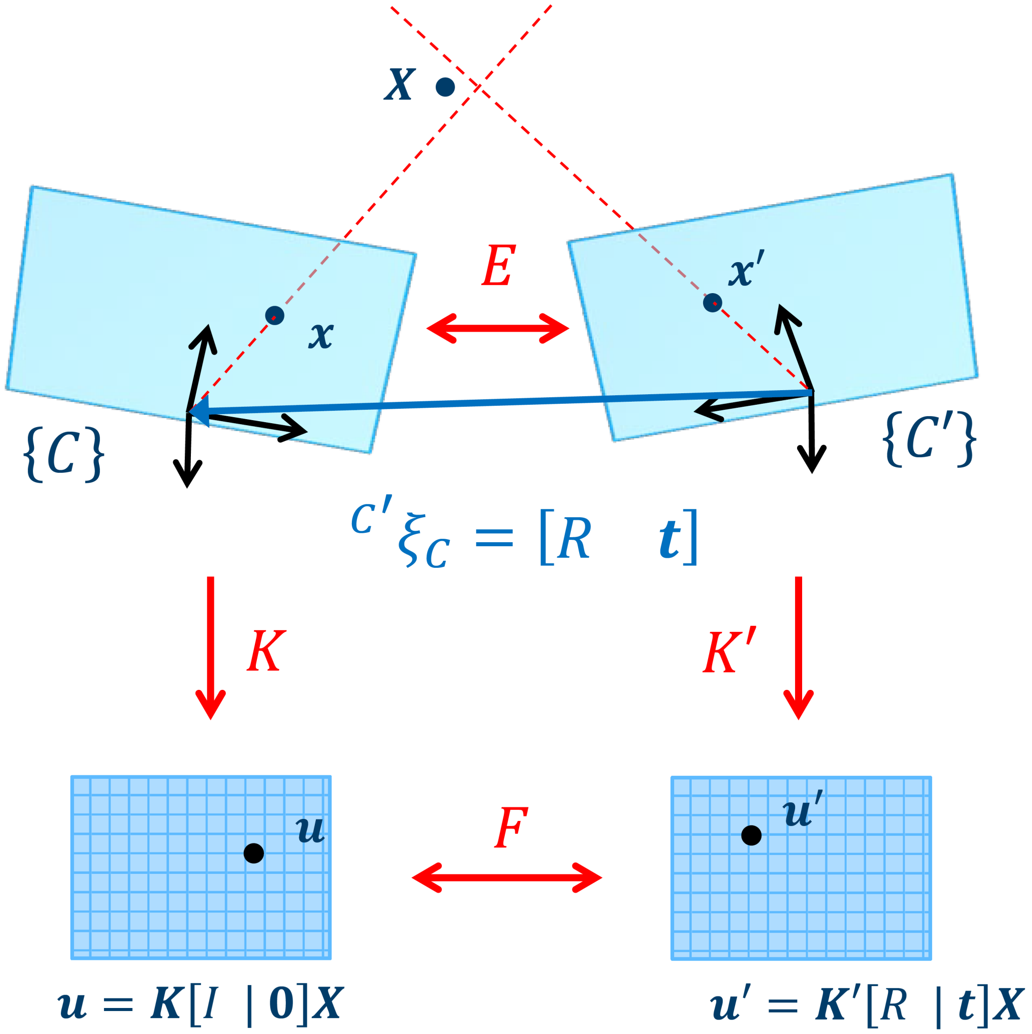
Matching Two Views

$$\mathrm{E} = [\mathbf{t}]_\times \mathrm{R}$$

$$\begin{aligned} E \cdot \mathbf{t} &= [\mathbf{t}]_\times R \cdot \mathbf{t} \\ &= (\mathbf{t} \times R) \cdot \mathbf{t} = 0 \end{aligned}$$

Use SVD to solve for **t**

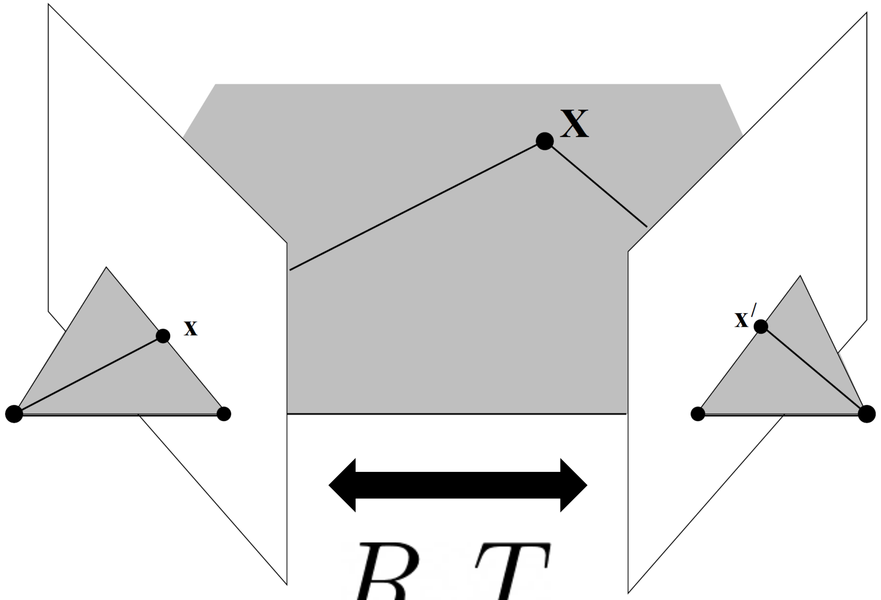
$$R = -[\mathbf{t}]_\times E$$



Credit: Thomas Opsahl

H. C Longuet-Higgins, *A computer algorithm for reconstructing a scene from two projections*, 1981

Triangulation



R, T

Estimated from essential matrix E

Intersection of two backprojected lines

$$\mathbf{X} = \mathbf{l} \times \mathbf{l}'$$

How to get the initial estimation β_0 ?

$$\beta = (\mathbf{X}, \mathbf{R}, \mathbf{T})$$

Structure from Motion

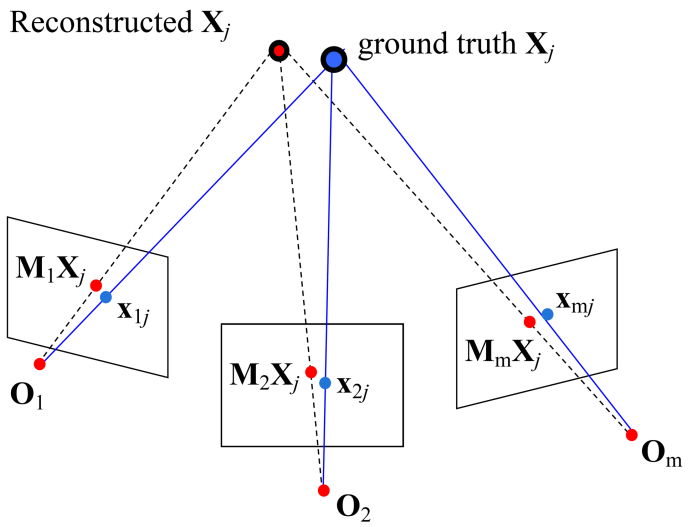
Bundle adjustment

- Iteratively refinement of structure (3D points) and motion (camera poses)

- Levenberg-Marquardt algorithm

$$g(\mathbf{X}, \mathbf{R}, \mathbf{T}) = \sum_{i=1}^{m} \sum_{j=1}^{n} \underbrace{w_{ij}} \cdot \left\| \underbrace{\mathbf{P}(\mathbf{x}_i, \mathbf{R}_j, \mathbf{t}_j)}_{\substack{\textit{predicted} \\ \text{image location}}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\substack{\textit{observed} \\ \text{image location}}} \right\|^2$$

indicator variable:
is point *i* visible in image *j* ?



Examples: http://vision.soic.indiana.edu/projects/disco/

Build Rome in One Day

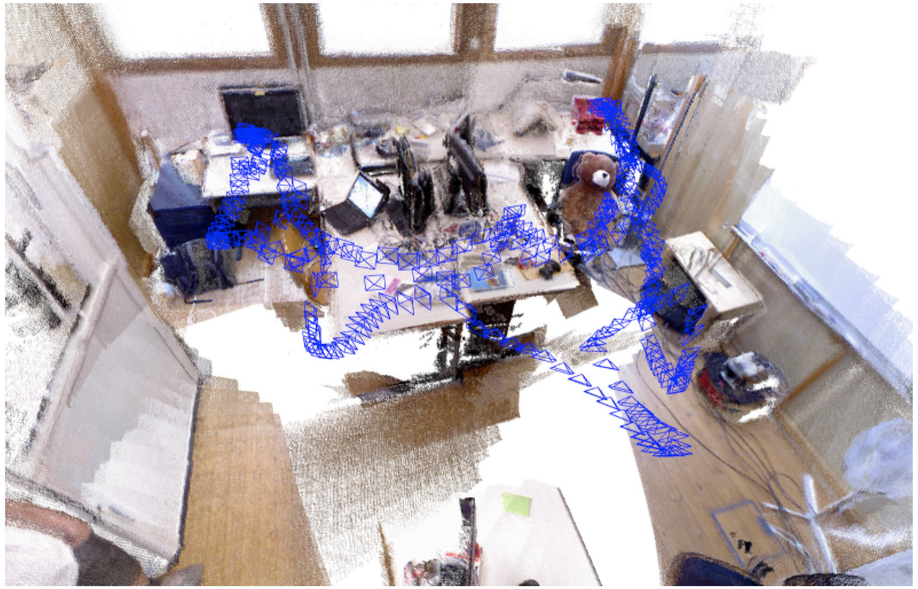
https://grail.cs.washington.edu/rome/

Simultaneous Localization and Mapping (SLAM)

Localization: camera pose tracking

Mapping: building a 2D or 3D representation of the environment

The goal here is the same as structure from motion but with video input

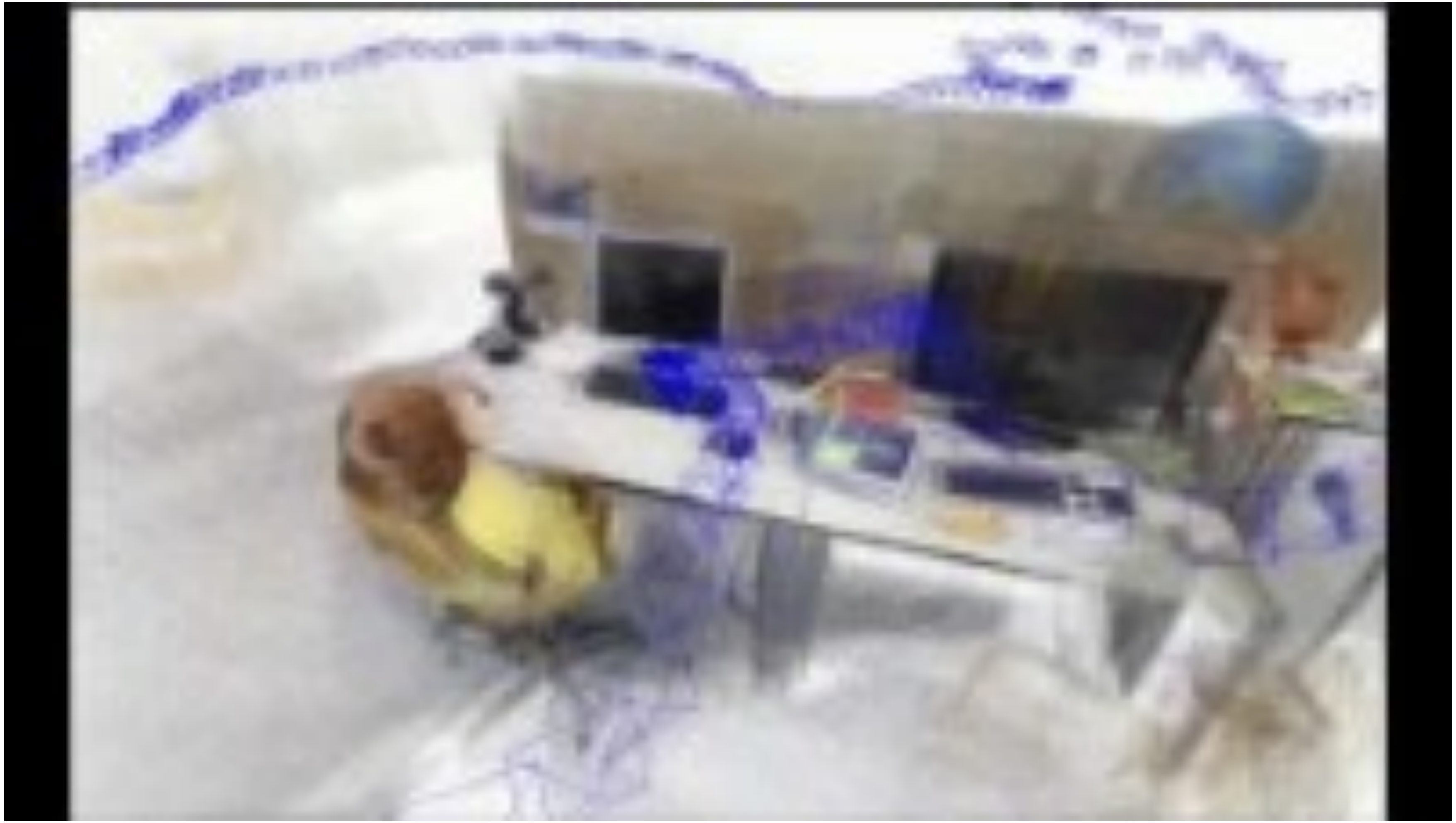


ORB-SLAM2

- Point cloud and camera poses

Case Study: ORB-SLAM

- Oriented FAST and Rotated BRIEF (ORB)
- Tracking camera poses
  - Motion only Bundle Adjustment (BA)
- Mapping
  - Local BA around camera pose (3D location refinement)
- Loop closing
  - Loop detection



https://webdiis.unizar.es/~raulmur/orbslam/

Case Study: ORB-SLAM

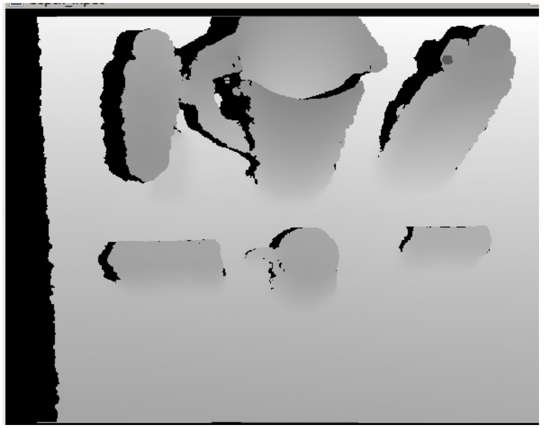
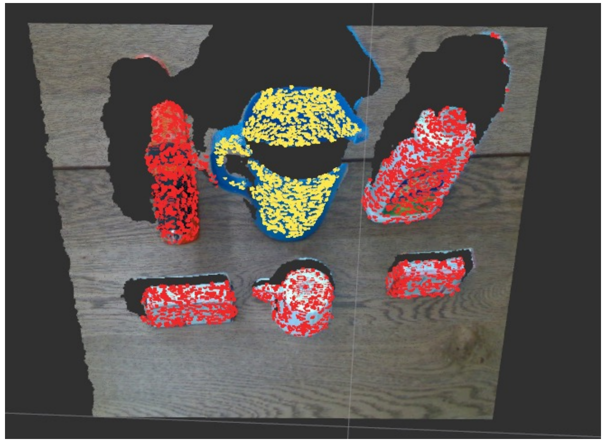
RGB-D SLAM

RGB-D cameras

Microsoft Kinect

Intel RealSense

Using depth images: 3D points in the camera frame

Point Cloud

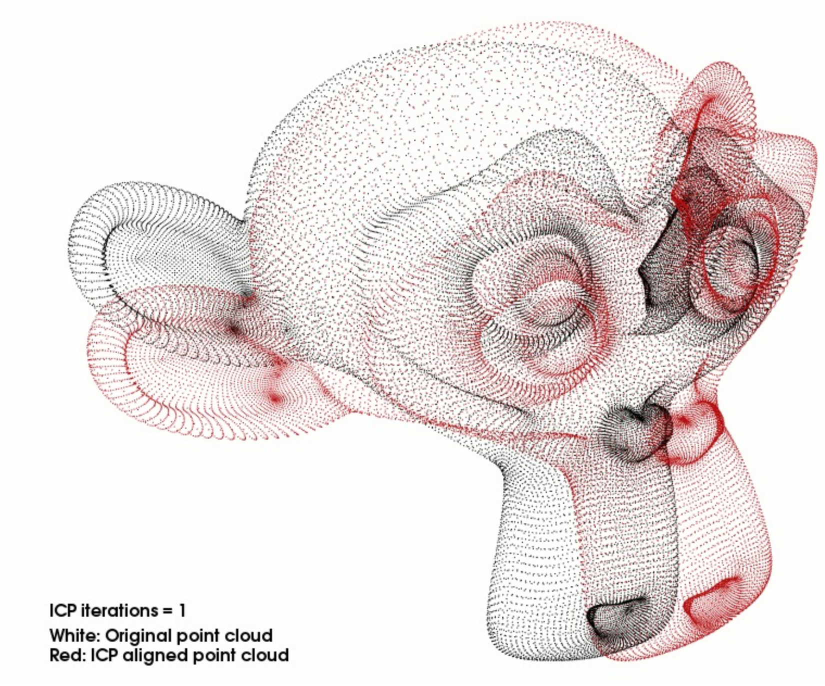
RGB-D SLAM

Camera pose tracking

- Iterative closest point (ICP) algorithm

Input: source point cloud, target point cloud
Output: rigid transformation from source to target

- For i in range(N)
  - For each point in the source, find the closest point in the target (correspondences)
  - Estimation R and T using the correspondences
  - Transform the source points using R and T



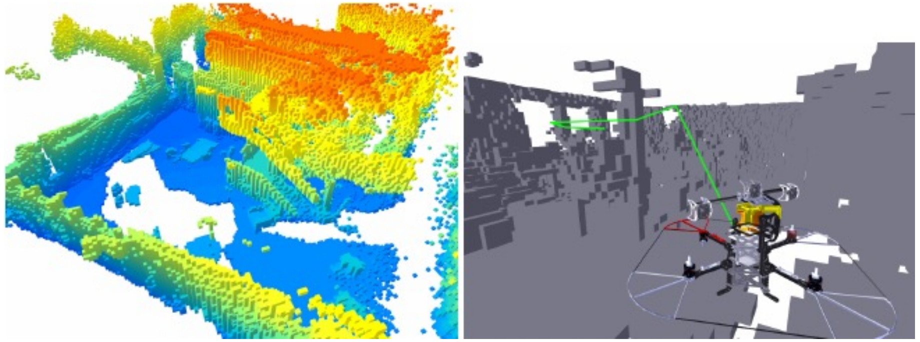
RGB-D SLAM

Mapping: fuse point clouds into a global frame

Map representation

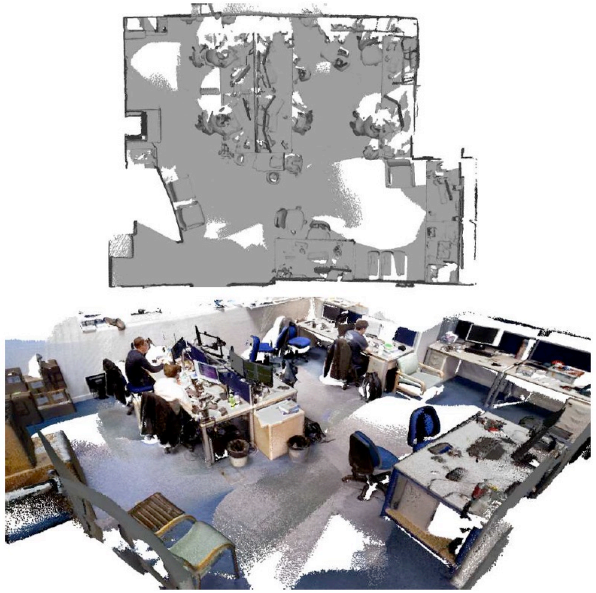
Point clouds

ORB-SLAM



Voxels

Visual Odometry and Mapping for Autonomous Flight Using an RGB-D Camera. Huang, et al. 2011



Surfels (small 3D surface)

ElasticFusion

KinectFusion

https://youtu.be/of6d7C_ZWwc

Further Reading

Chapter 11, Computer Vision, Richard Szeliski

KinectFusion: Real-Time Dense Surface Mapping and Tracking. Newcombe et al., ISMAR'11

ORB-SLAM https://webdiis.unizar.es/~raulmur/orbslam/