



Camera Models

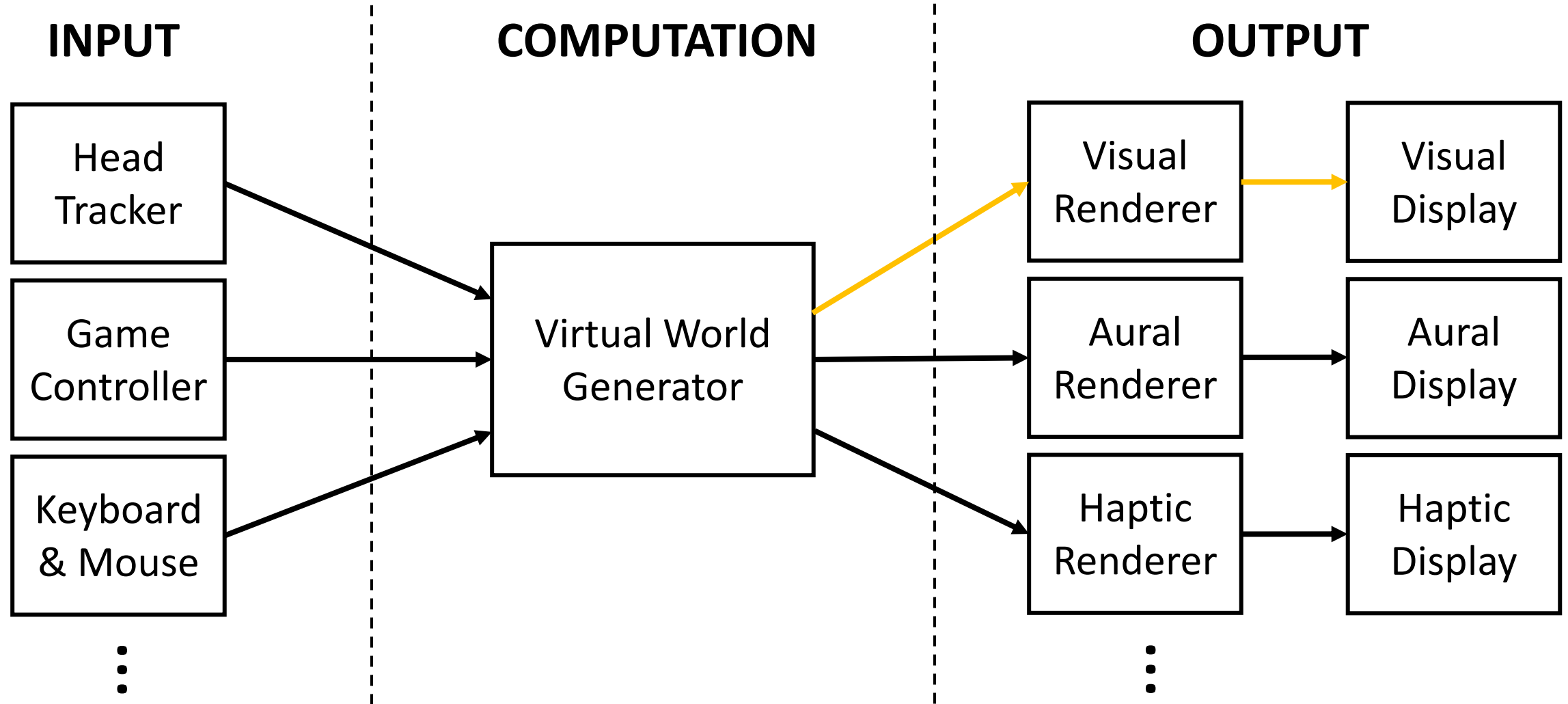
CS 6334 Virtual Reality

Professor Yapeng Tian

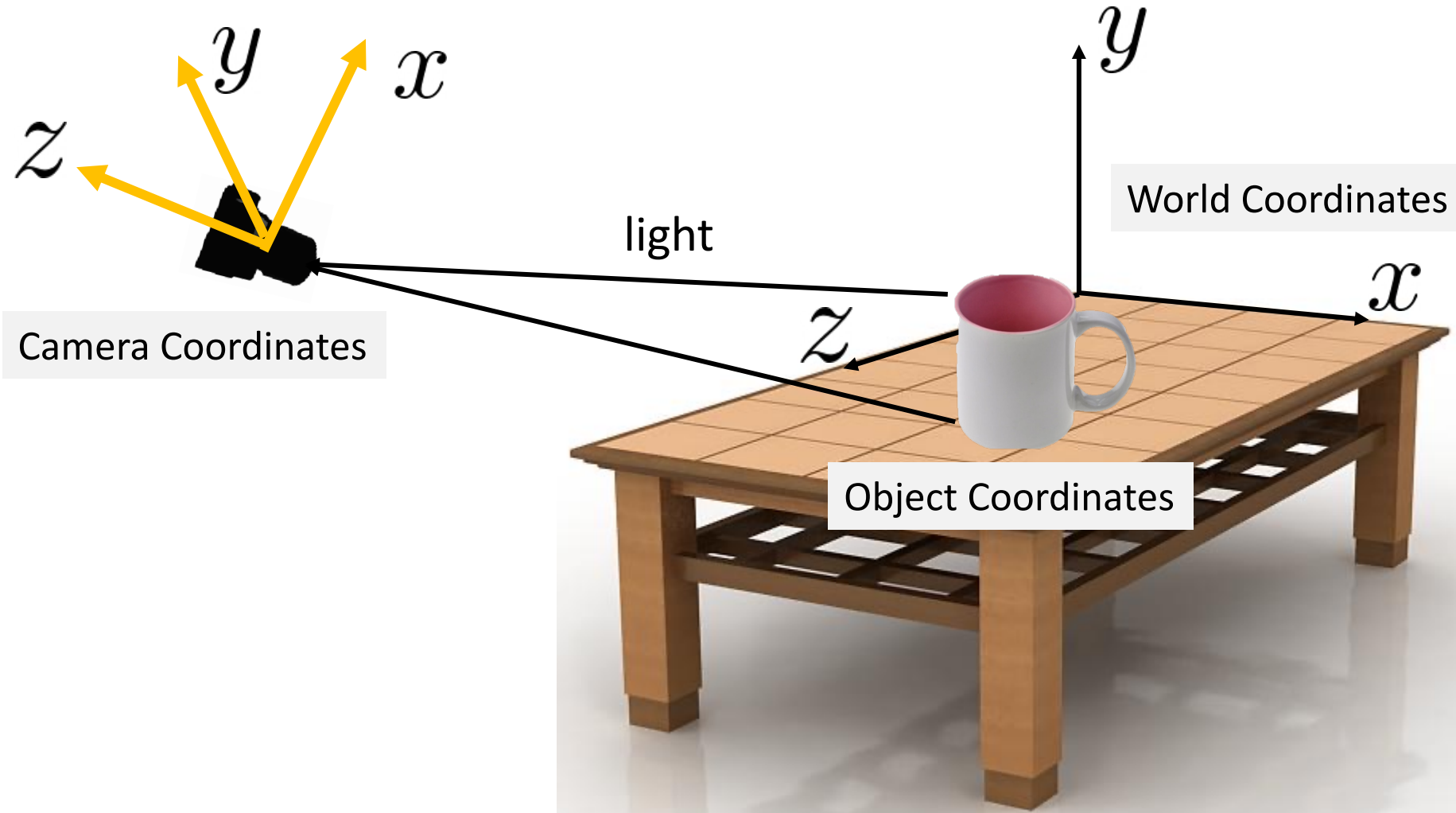
The University of Texas at Dallas

A lot of slides of course lectures borrowed from Professor Yu Xiang's VR class

Review of VR Systems



A Virtual World with a Camera



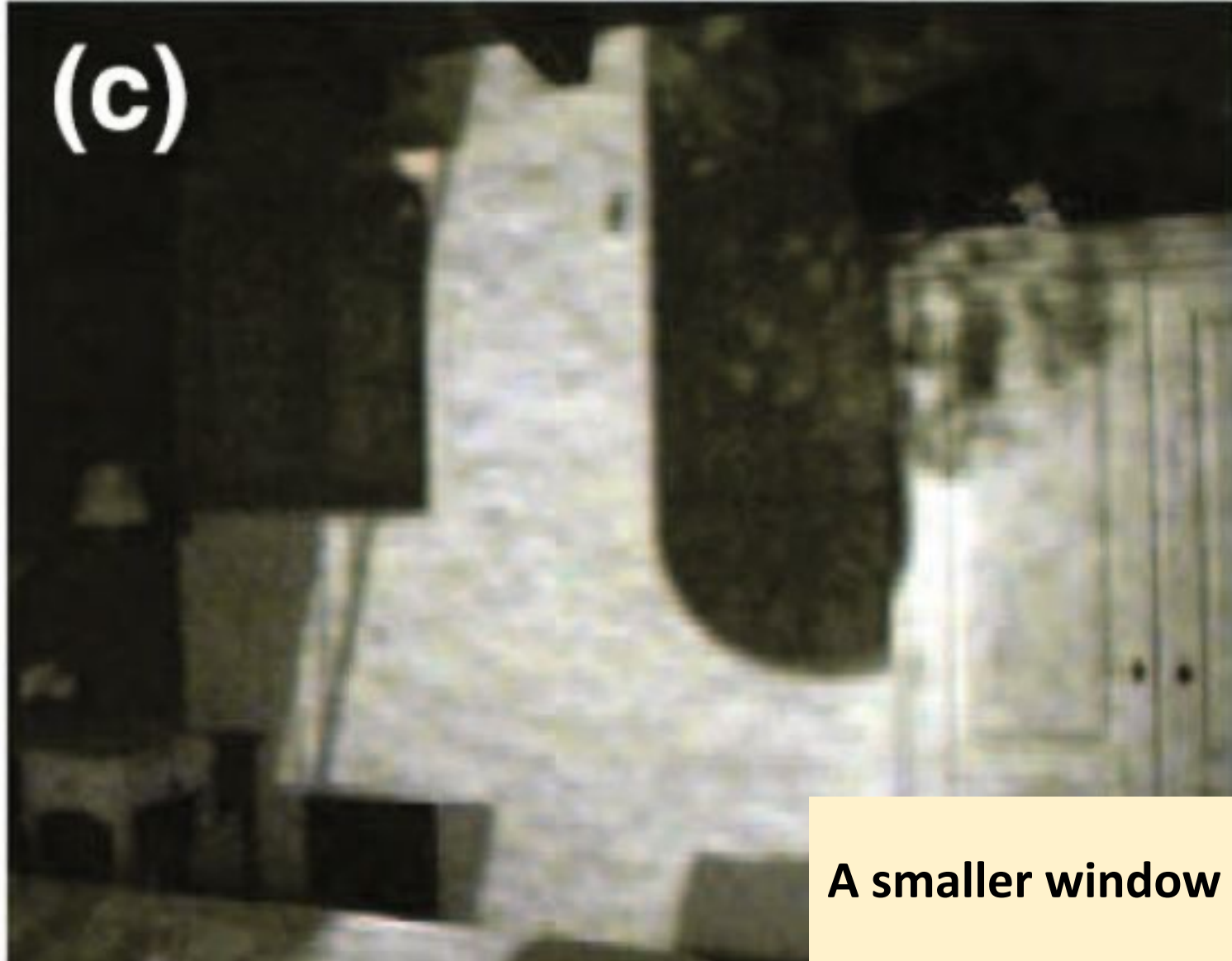


A Living Room

What objects or scenes?

Largely opened window

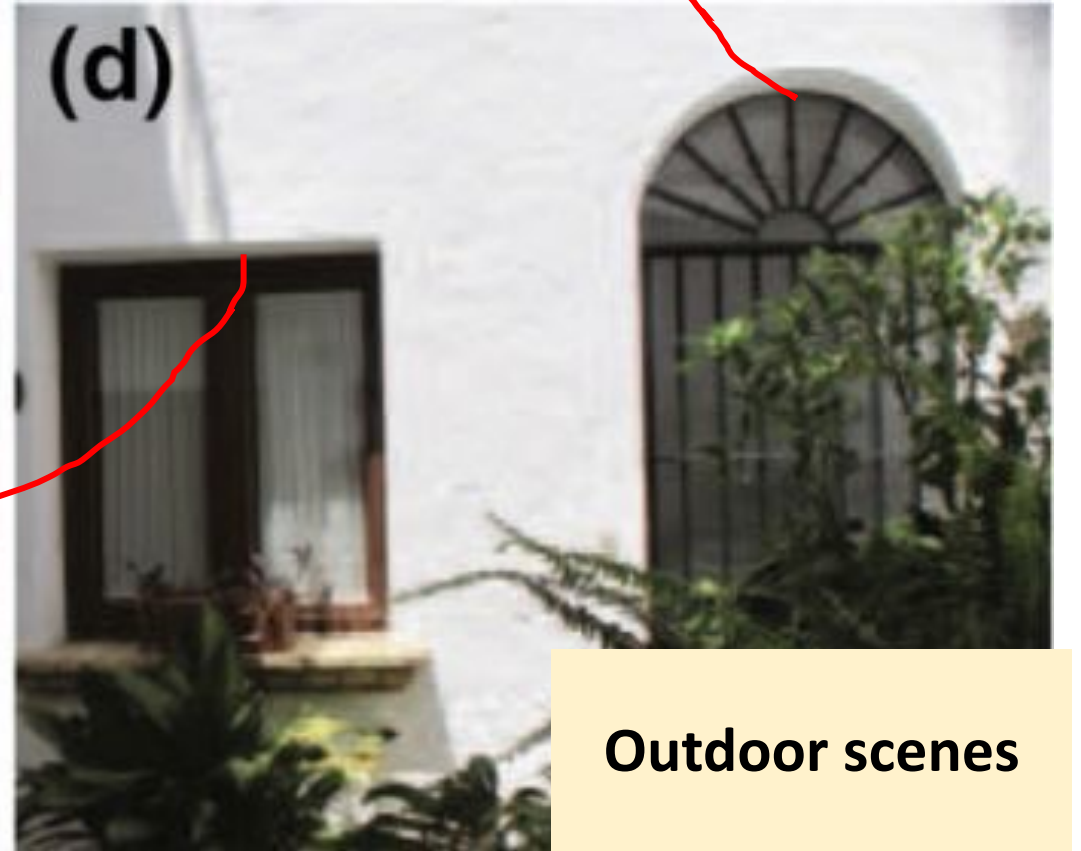
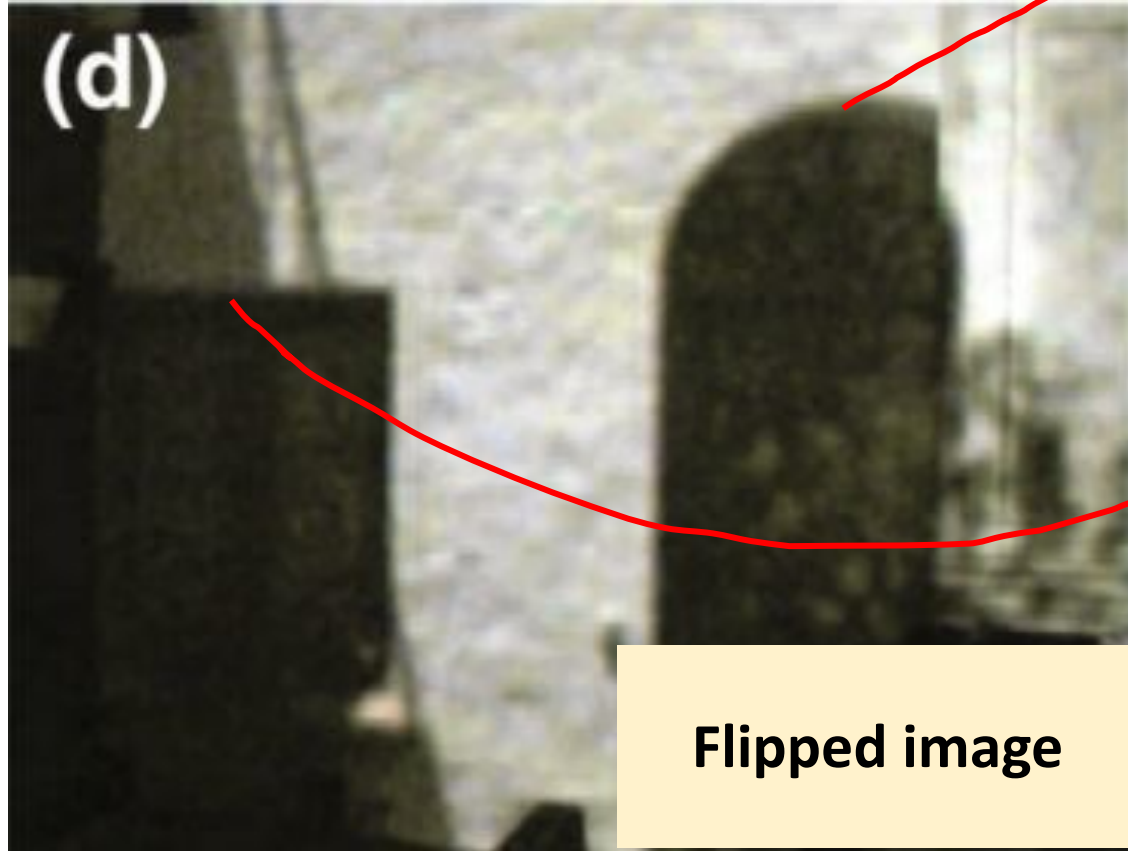




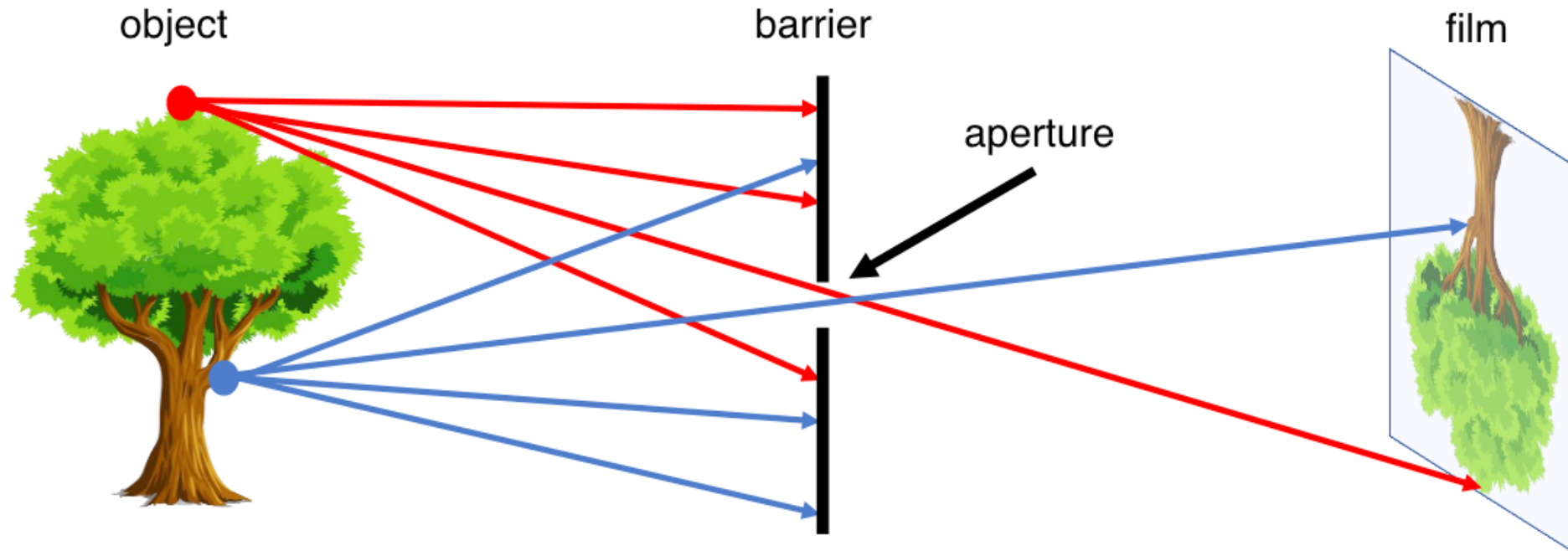
(c)

A smaller window

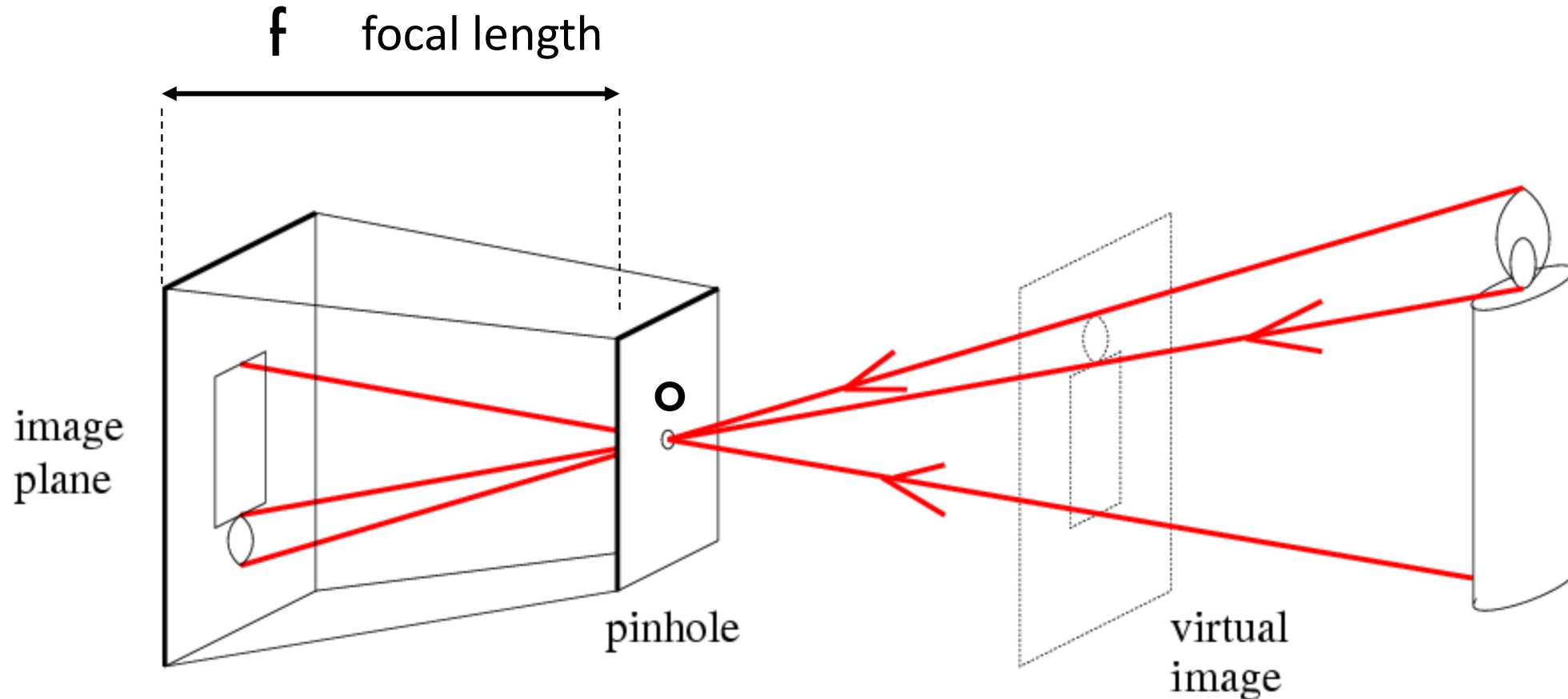
Nature Example of Pinhole Camera



Pinhole Camera



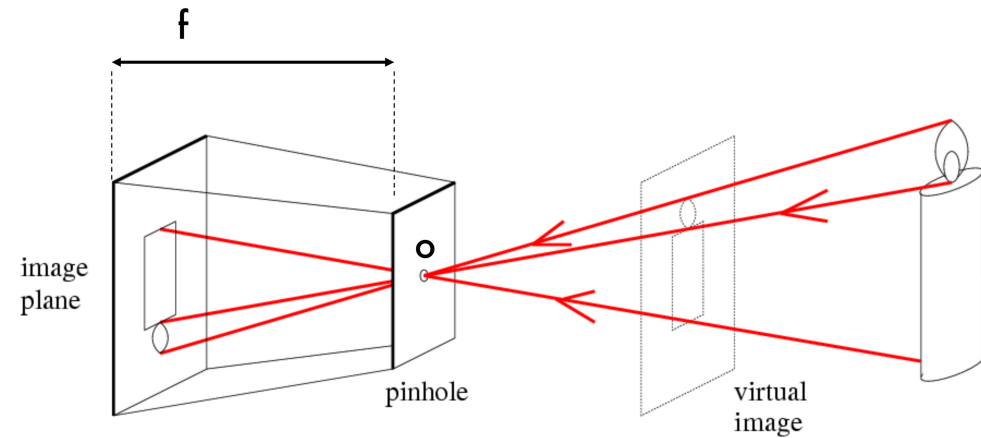
Pinhole Camera



Rotate the image plane by 180°

Cannot be implemented in practice
Useful for theoretic analysis

Natural Pinhole Cameras

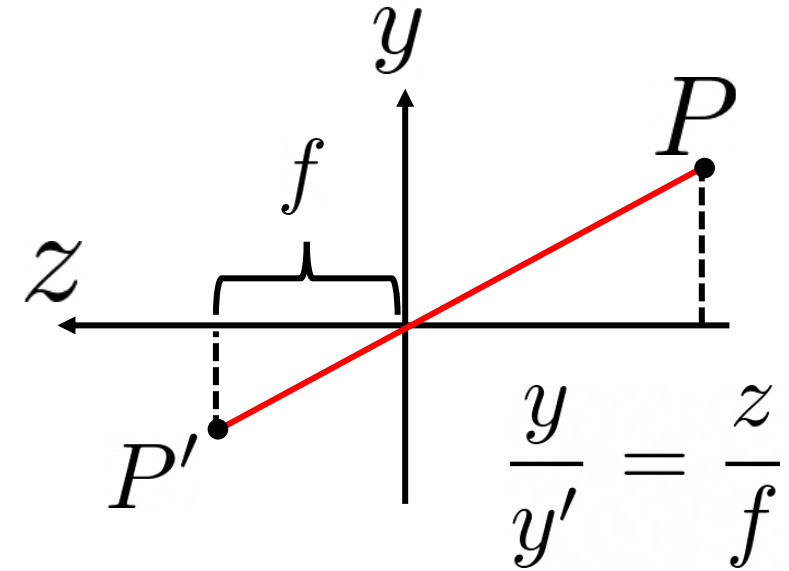
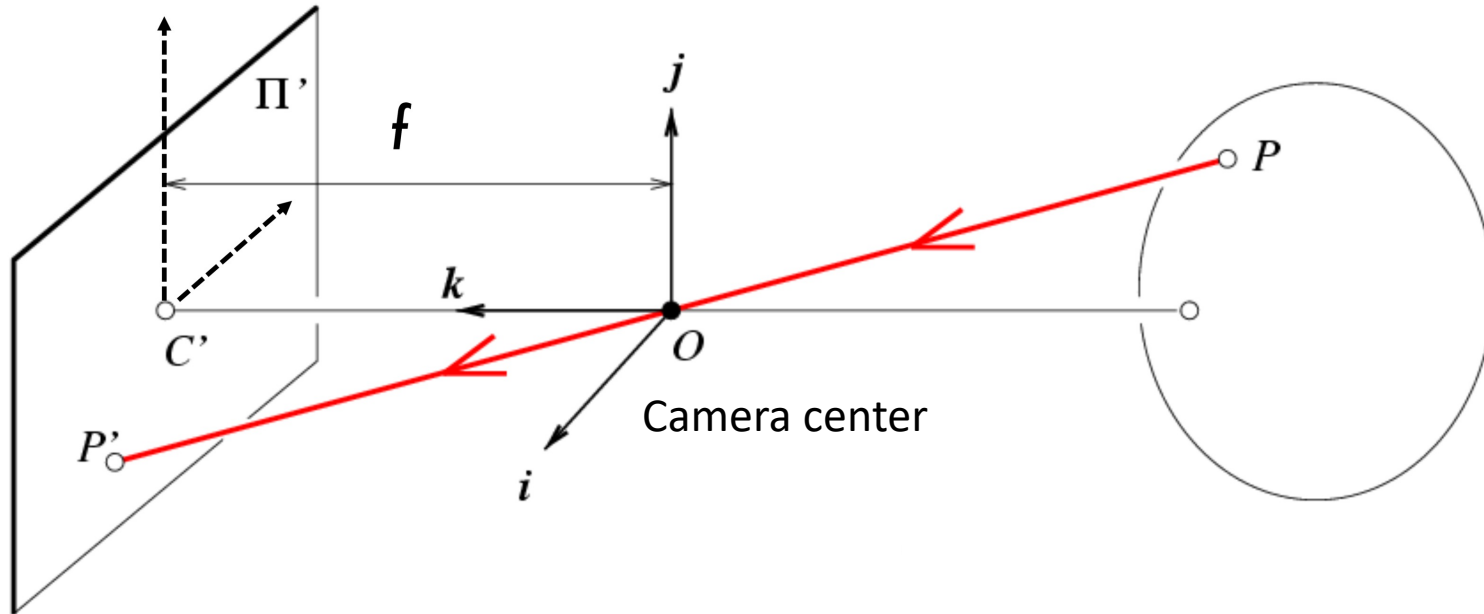


Object: the sun

Pinhole: gaps between the leaves

Image plane: the ground

Central Projection in Camera Coordinates



Camera coordinates

$$\mathbf{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{\text{Nonlinear}} \mathbf{P}' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f \frac{x}{z} \\ y' = f \frac{y}{z} \end{cases}$$

Homogeneous Coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image
coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene
coordinates

Conversion

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Central Projection with Homogeneous Coordinates

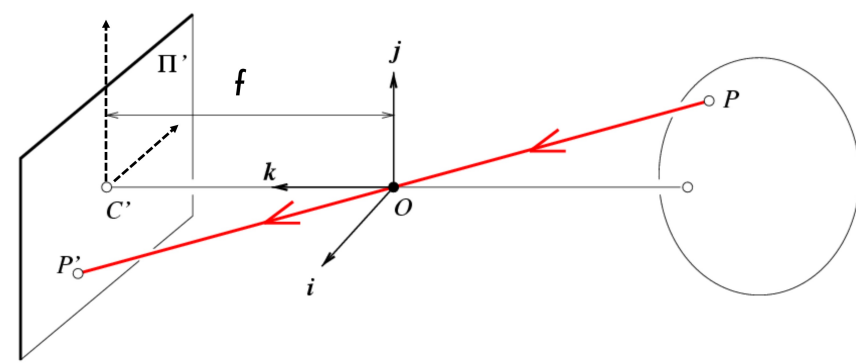
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} f \frac{x}{z} \\ f \frac{y}{z} \\ z \end{bmatrix}$$

Central projection

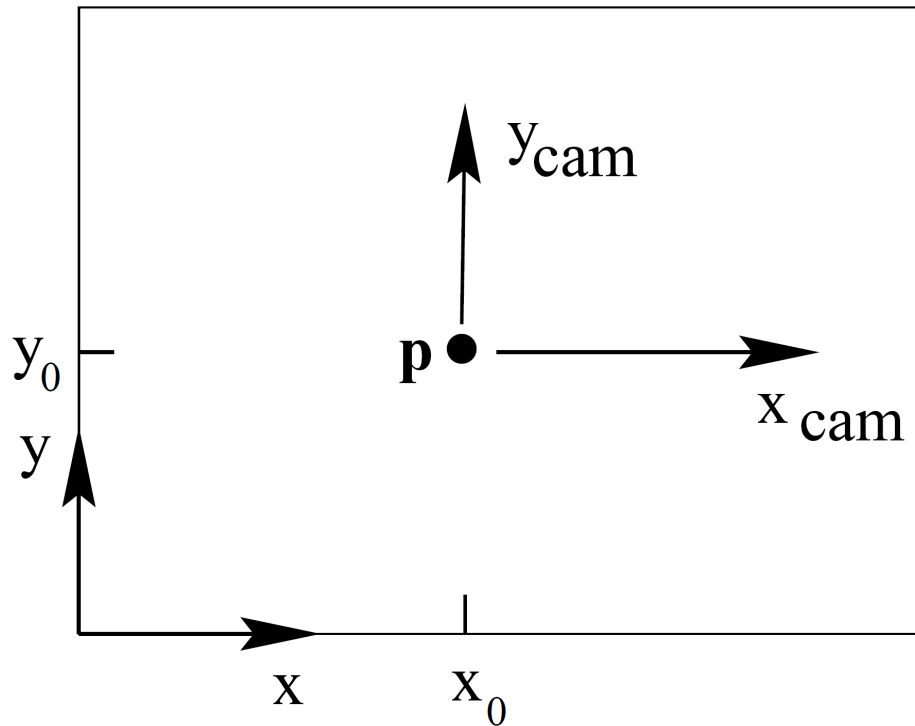
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3x4 matrix

Principal Point Offset



Principal point $\mathbf{p} = (p_x, p_y)$

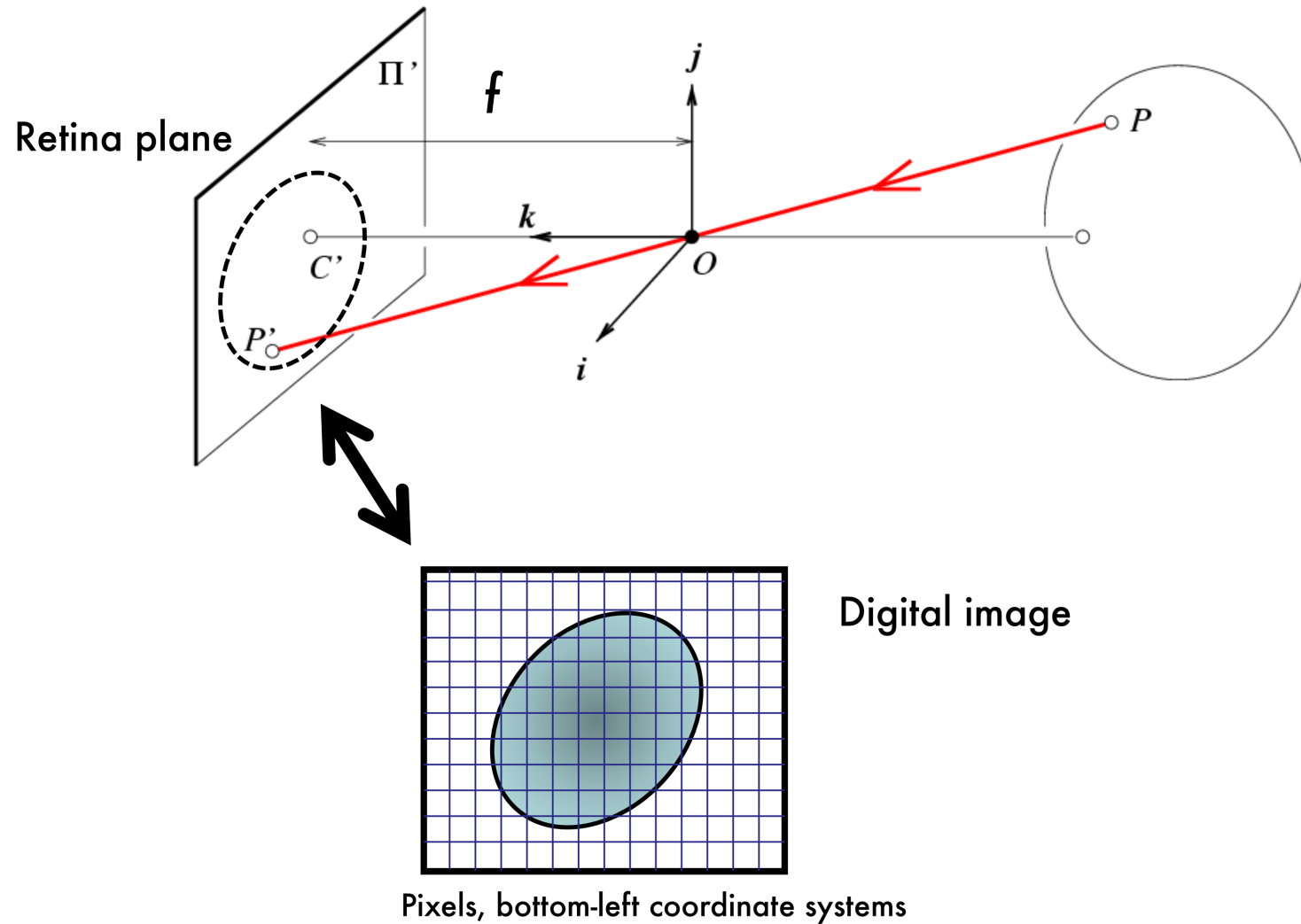


Principle point: projection of the camera center

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} f \frac{x}{z} + p_x \\ f \frac{y}{z} + p_y \end{bmatrix}$$

$$\begin{bmatrix} f & p_x & 0 \\ & f & p_y & 0 \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

From Metric to Pixels



From Metric to Pixels

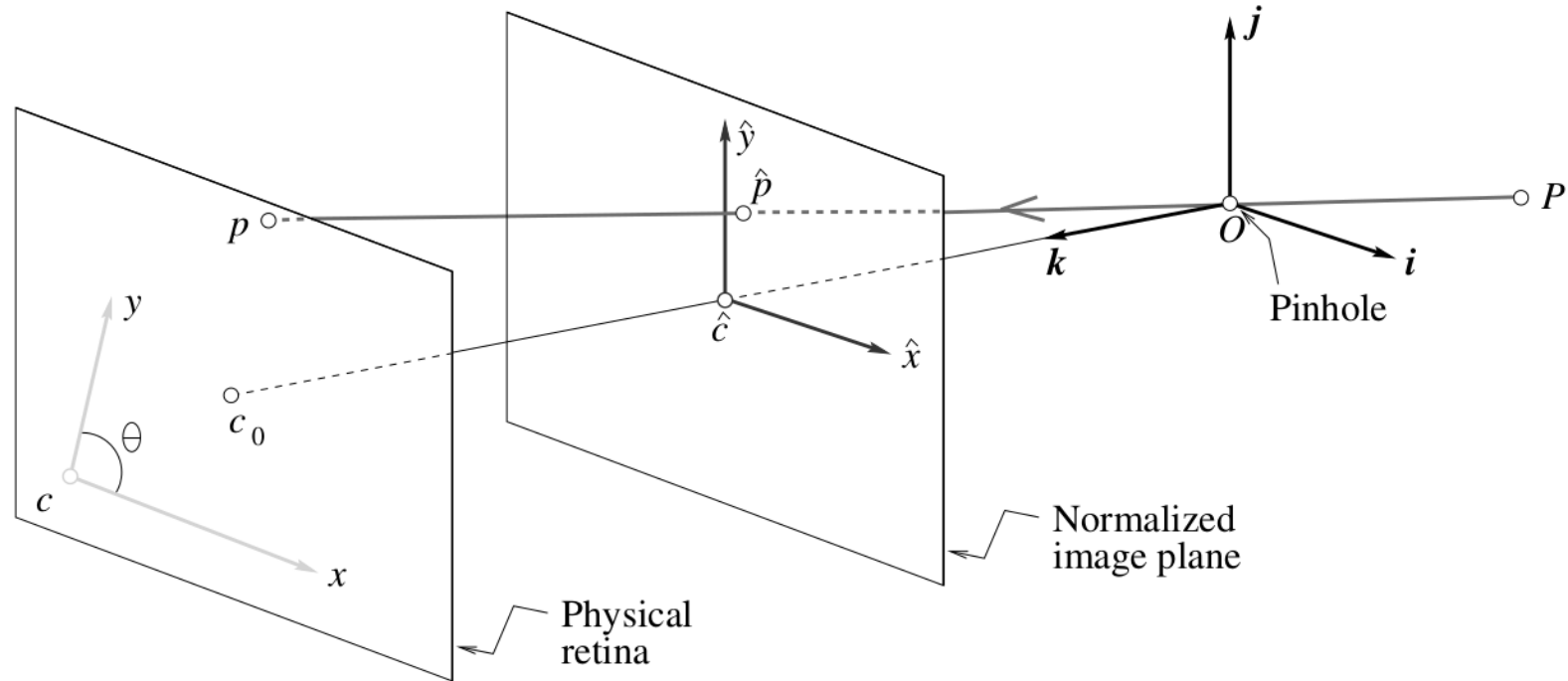
- Metric space, i.e., meters
$$\begin{bmatrix} f & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix}$$

- Pixel space
$$\begin{bmatrix} \alpha_x & x_0 & 0 \\ & \alpha_y & y_0 & 0 \\ & & 1 & 0 \end{bmatrix}$$

$$\alpha_x = f m_x$$
$$\alpha_y = f m_y$$
$$x_0 = p_x m_x$$
$$y_0 = p_y m_y$$

m_x, m_y Number of pixel per unit distance

Axis Skew



The skew parameter will be zero for most normal cameras.

$$\begin{bmatrix} \alpha_x & & x_0 & 0 \\ & \alpha_y & y_0 & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \alpha_x \frac{x}{z} + x_0 \\ \alpha_y \frac{y}{z} + y_0 \end{bmatrix} \quad \begin{bmatrix} \alpha_x & -\alpha_x \cot(\theta) & x_0 & 0 \\ & \frac{\alpha_y}{\sin(\theta)} & y_0 & 0 \\ & & 1 & 0 \end{bmatrix}$$

<https://blog.immenselyhappy.com/post/camera-axis-skew/>

Camera Intrinsics

$$\begin{bmatrix} \alpha_x & -\alpha_x \cot(\theta) & x_0 & 0 \\ & \frac{\alpha_y}{\sin(\theta)} & y_0 & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

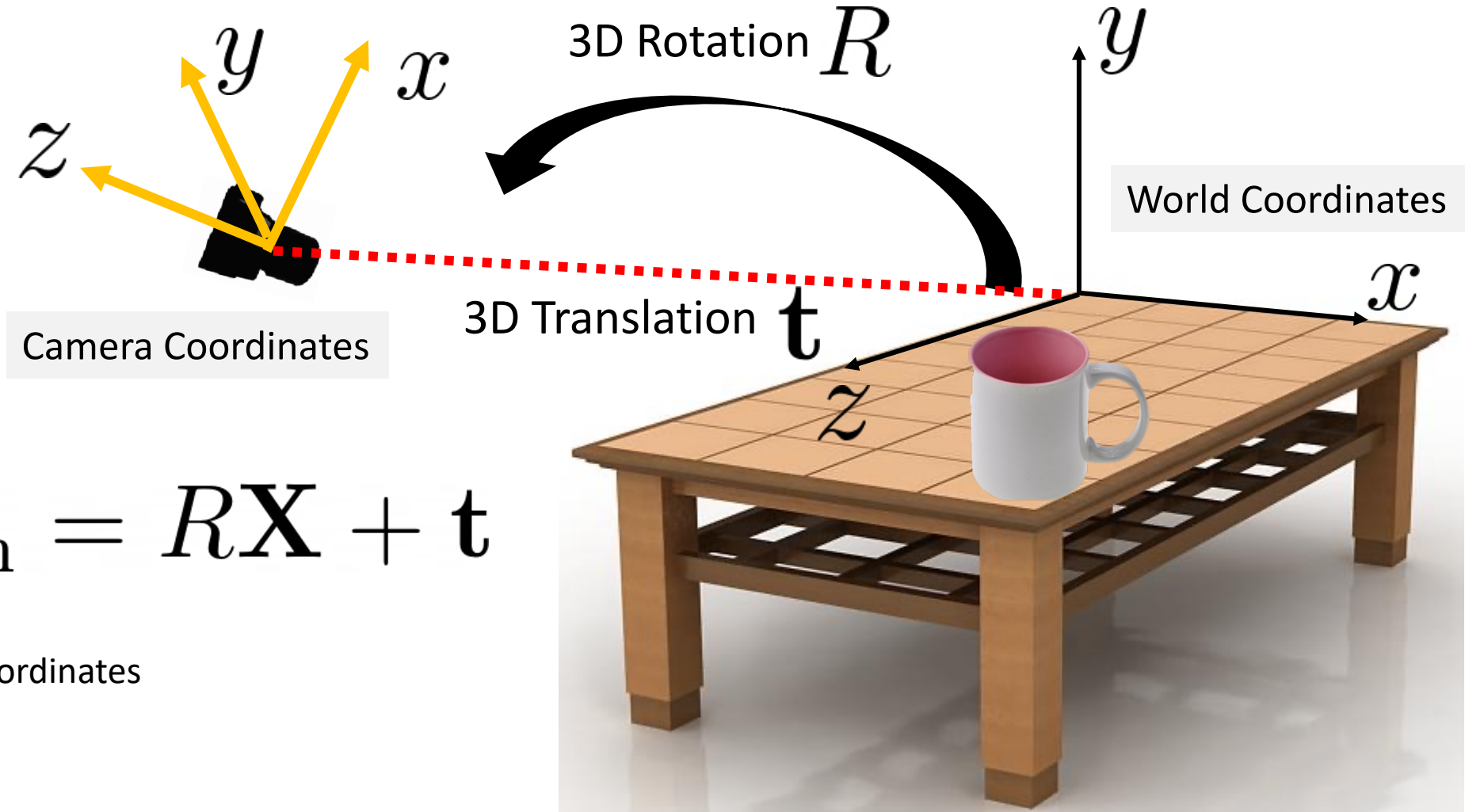
Camera intrinsics

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix} \quad \mathbf{X} = K [I | \mathbf{0}] \mathbf{X}_{\text{cam}}$$

3×1 3×3 3×4 4×1

Homogeneous coordinates

Camera Extrinsics: Camera Rotation and Translation



$$\mathbf{X}_{\text{cam}} = R\mathbf{X} + \mathbf{t}$$

Euclidean coordinates

Camera Projection Matrix $P = K[R|\mathbf{t}]$

- Homogeneous coordinates

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\text{cam}} \quad K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

$$= K[R|\mathbf{t}]\mathbf{X}$$

3x1

3x3

3x4

4x1

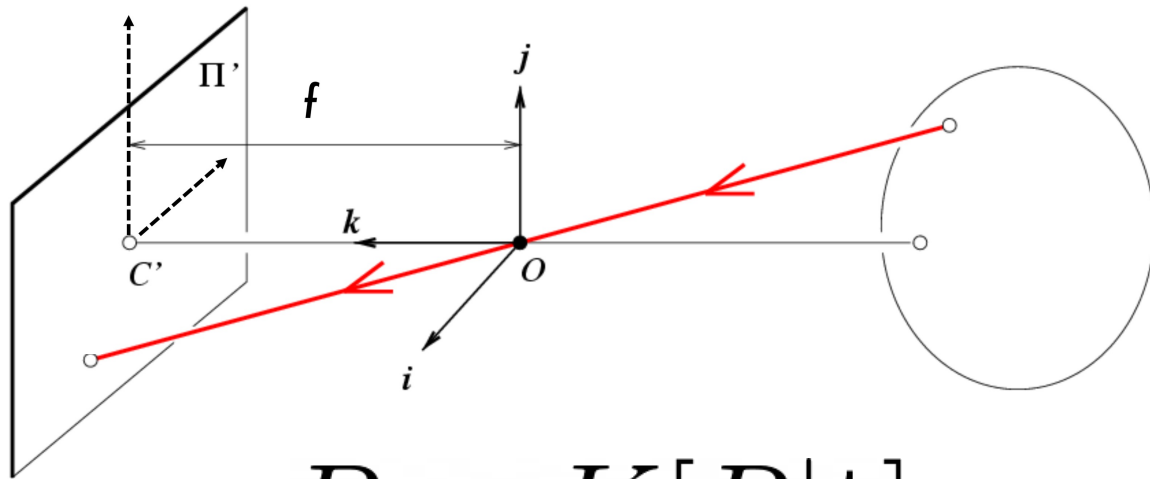
Image coordinates

Camera intrinsics

Camera extrinsics:
rotation and translation

World coordinates

Back-projection in World Coordinates



$$P = K[R|\mathbf{t}]$$

$$\mathbf{x} = P\mathbf{X}$$

- The camera center O is on the ray

- $P^+ \mathbf{x}$ is on the ray

$$P^+ = P^T (PP^T)^{-1}$$

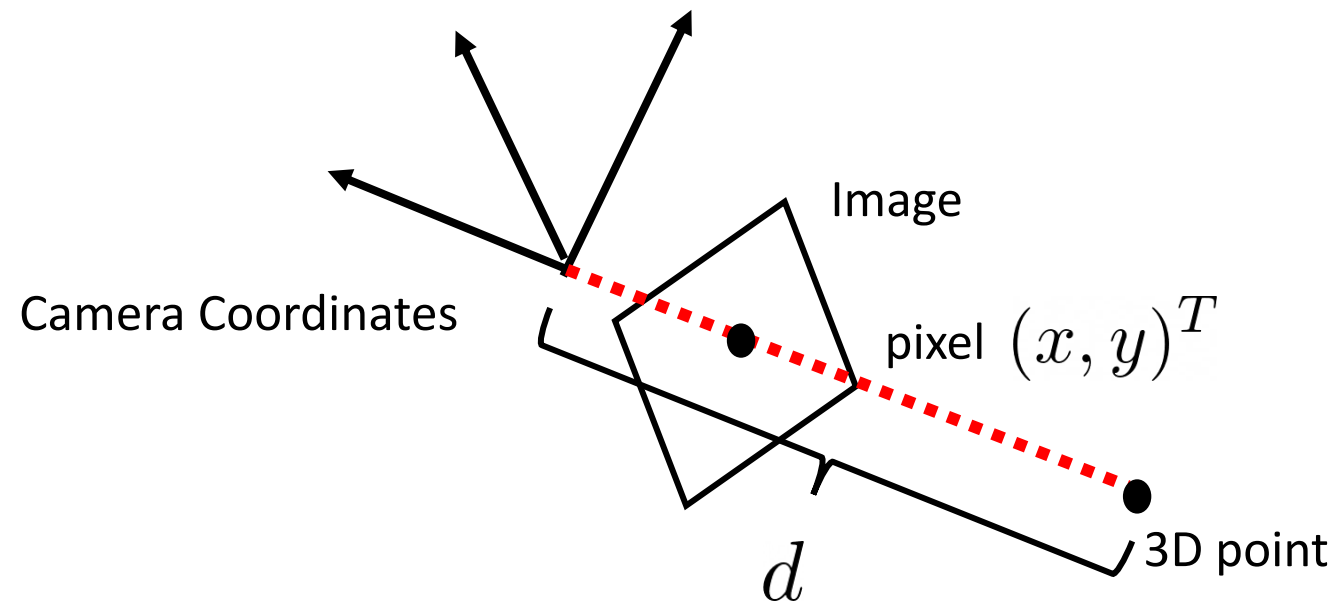
Pseudo-inverse

The ray can be written as

$$P^+ \mathbf{x} + \lambda O$$

- A pixel on the image backprojects to a ray in 3D

Back-projection in Camera Coordinates



$$P = K[I|\mathbf{0}]$$

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\text{cam}}$$

$$K^{-1}\mathbf{x}$$

3D point with depth d : $dK^{-1}\mathbf{x}$

3D camera coordinates
$$\begin{bmatrix} d \frac{x - p_x}{f_x} \\ d \frac{y - p_y}{f_y} \\ d \end{bmatrix}$$

Summary: Camera Models

- Camera projection matrix: intrinsics and extrinsics

$$P = K[R|\mathbf{t}]$$

3x3

3x4

Camera intrinsics

Camera extrinsics:
rotation and translation

Further Reading

- Stanford CS231A: Computer Vision, From 3D Reconstruction to Recognition, [Course Notes 1: Camera Models](#)
- [Multiview Geometry in Computer Vision](#), Richard Hartley and Andrew Zisserman, Chapter 6, Camera Models