THE UNIVERSITY OF TEXAS AT DALLAS

Image Formulation: Camera Models

CS 4391 Computer Vision

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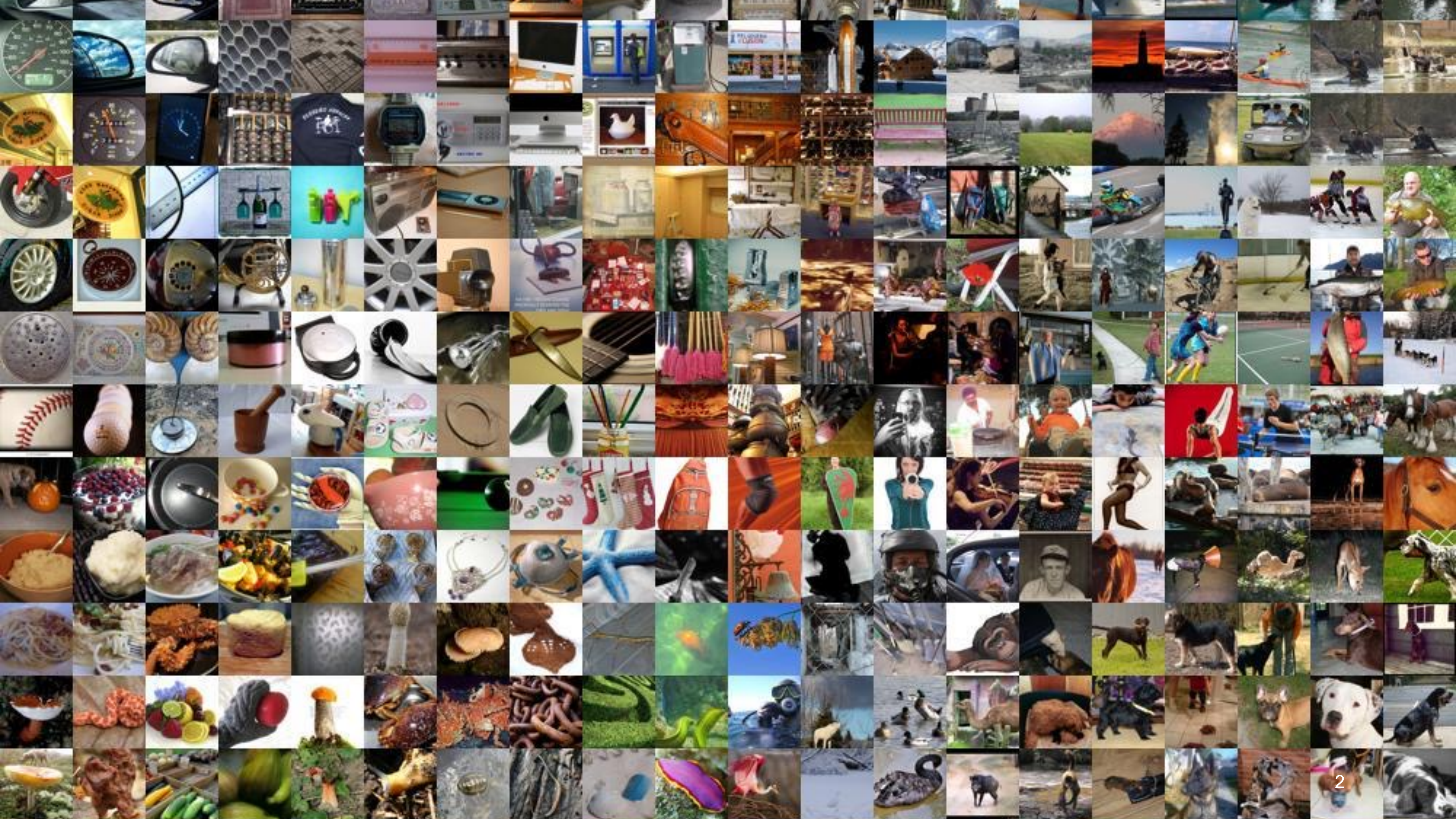
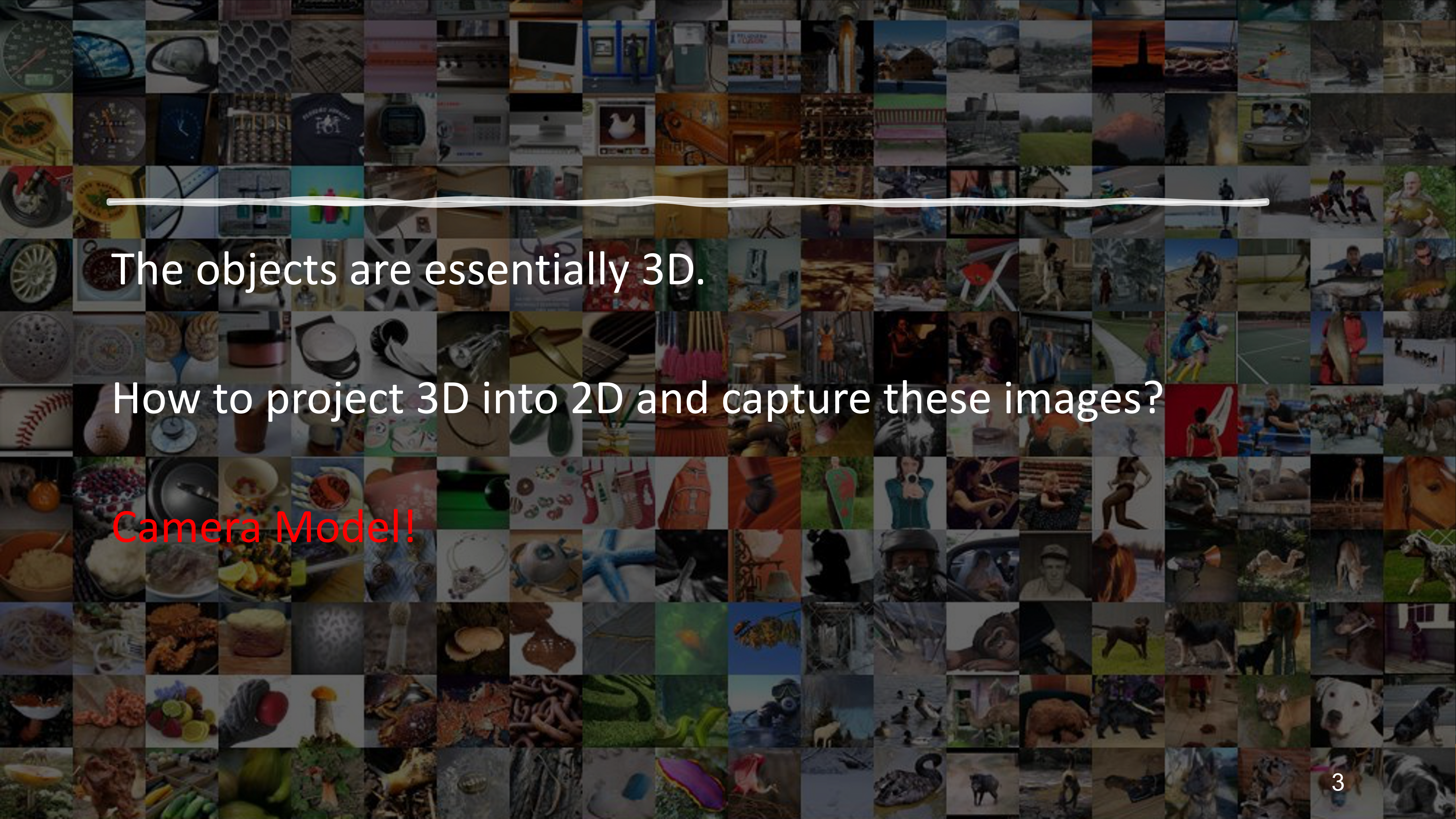
Department of Computer Science



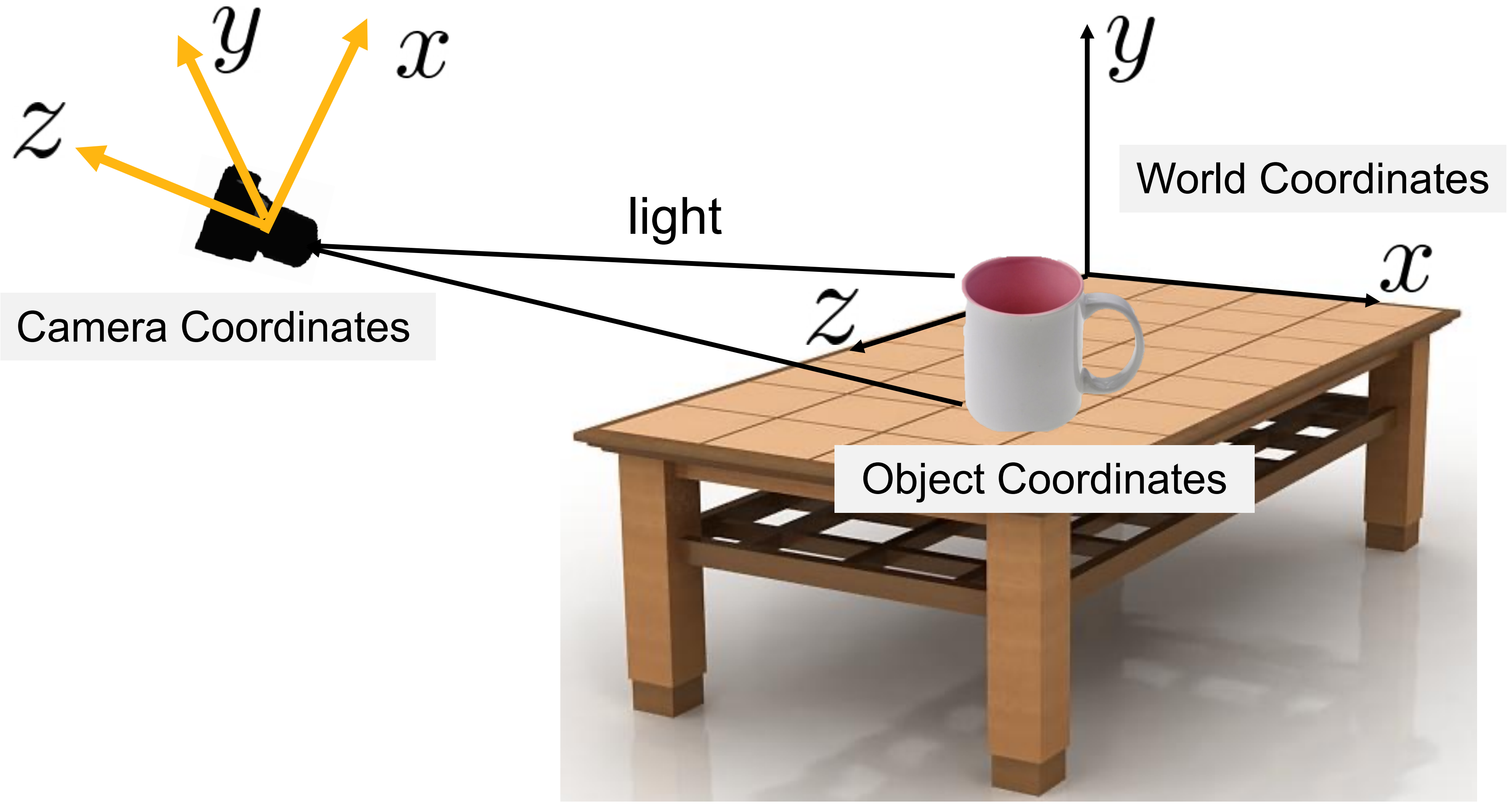


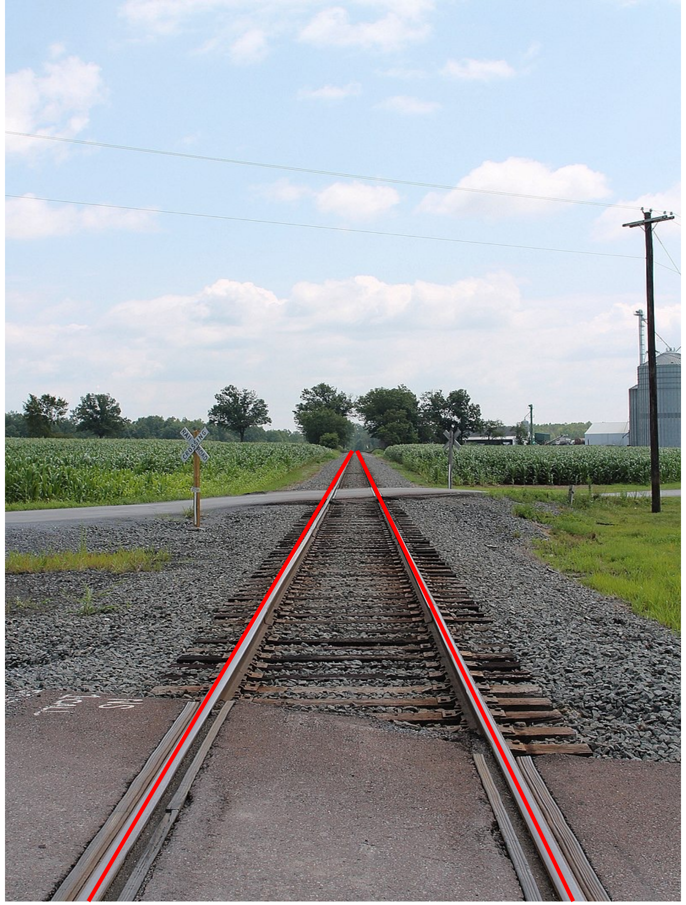
The objects are essentially 3D.

How to project 3D into 2D and capture these images?

Camera Model!

Camera Models: 3D-to-2D Projection





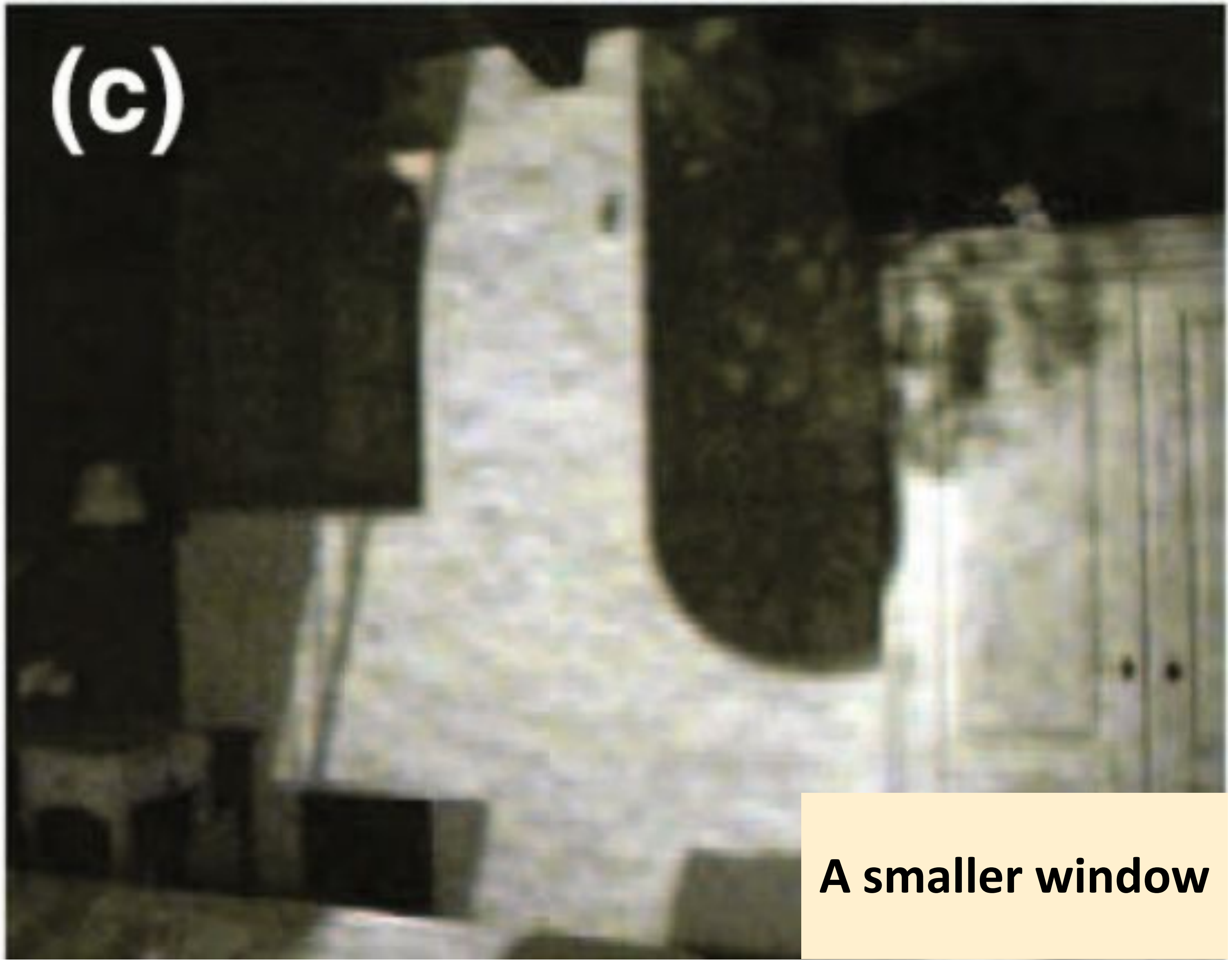
Q1: Are three balls in a same size?

Q2: Are the two rail lines parallel?

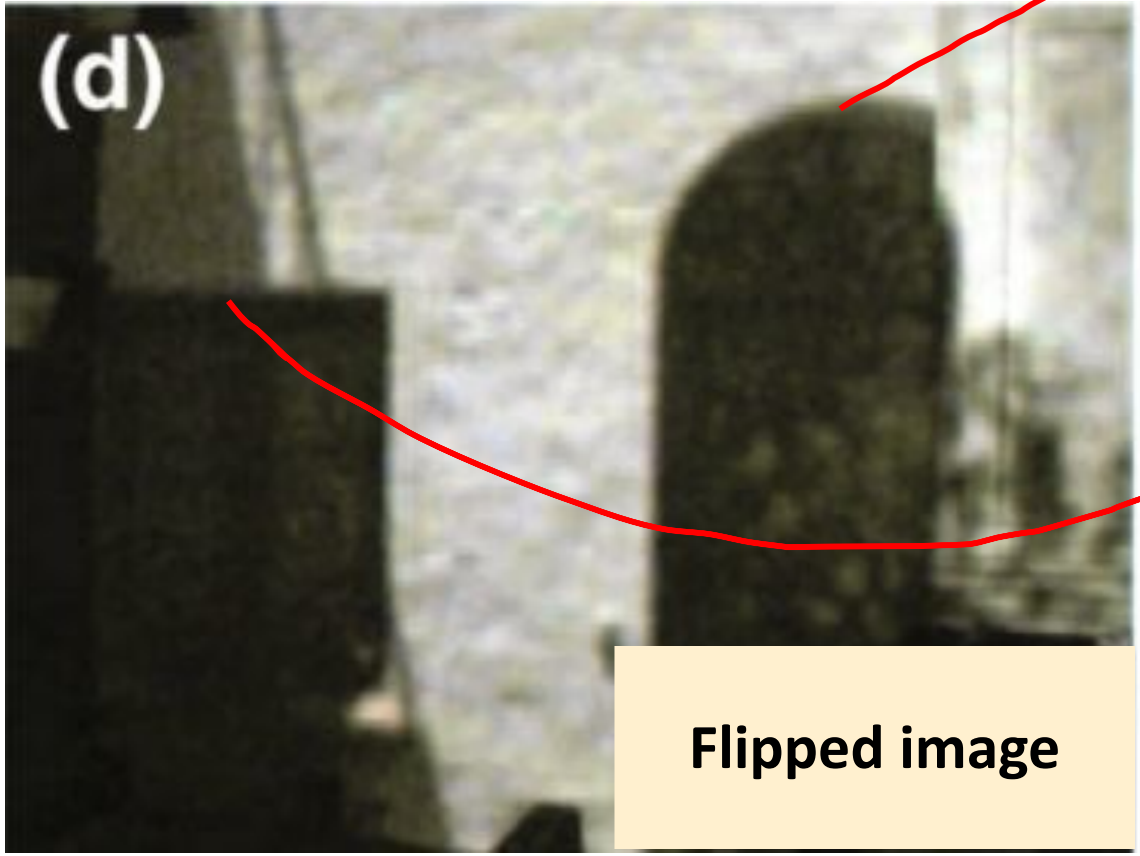
A1&A2: No?

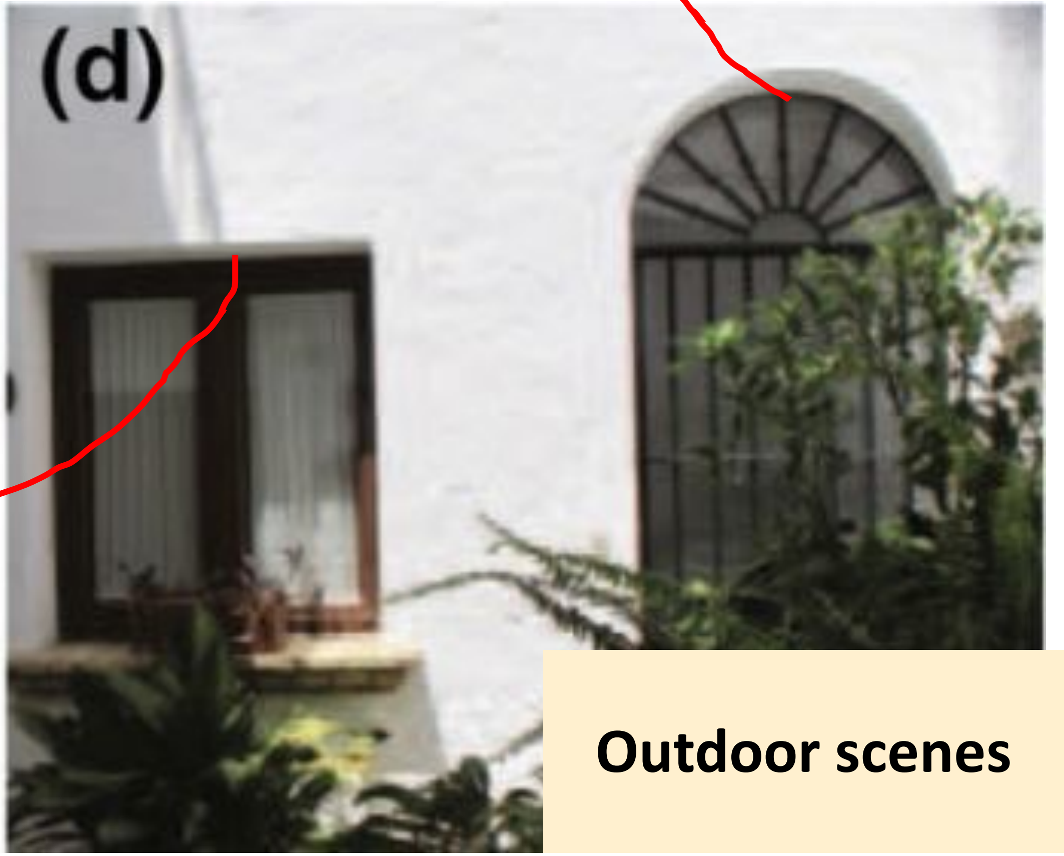
A Living Room
What objects or scenes?

Largely opened window

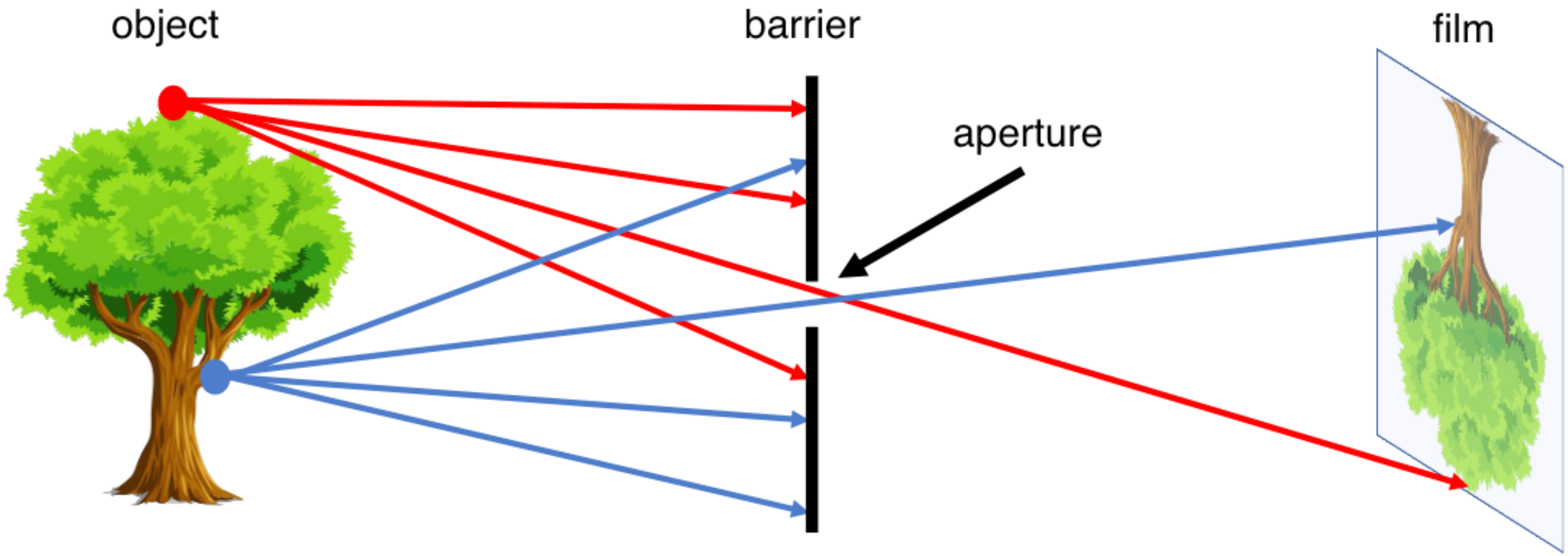

(c)
A smaller window

Nature Example of Pinhole Camera

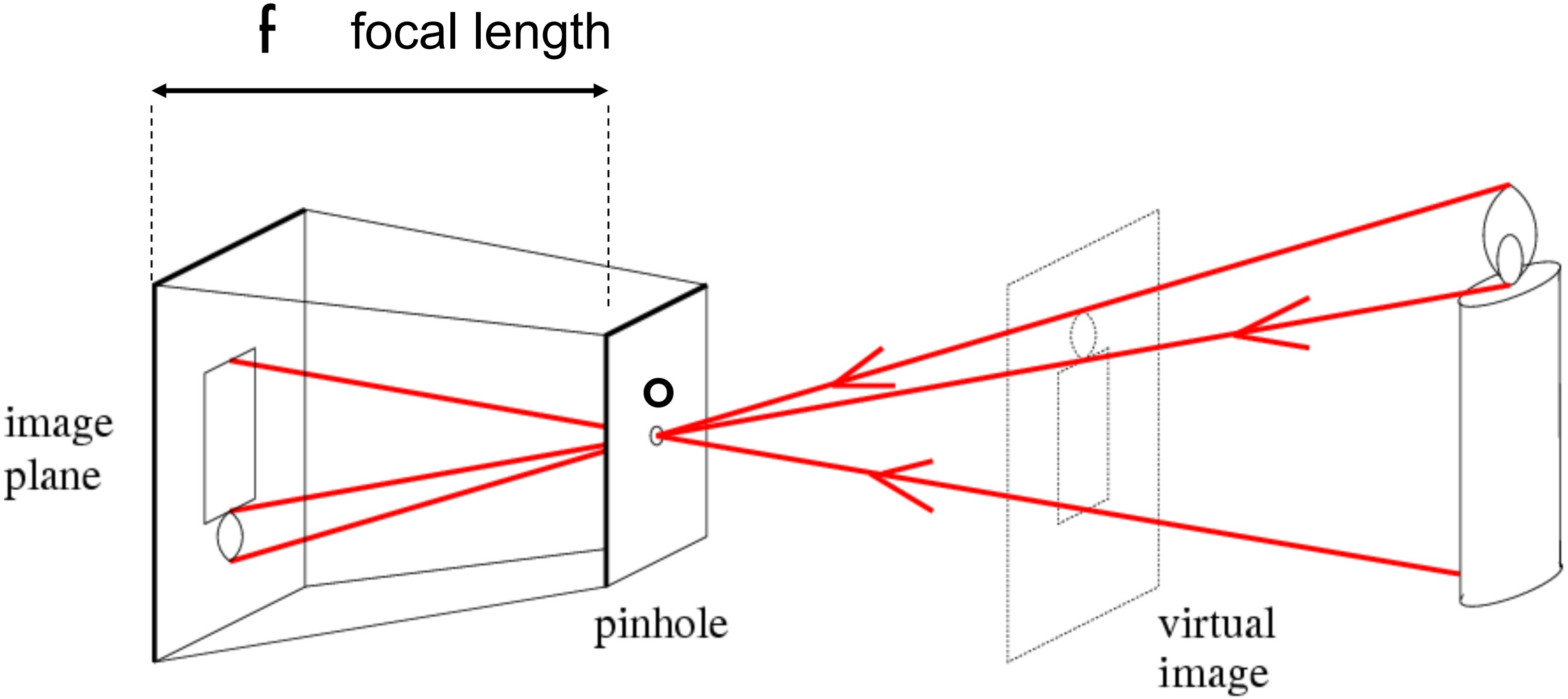




Pinhole Camera



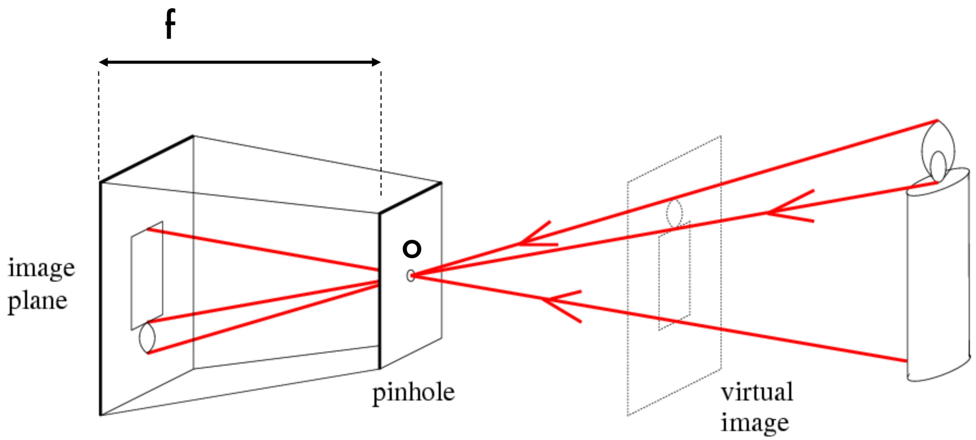
Pinhole Camera



Rotate the image plane by 180°

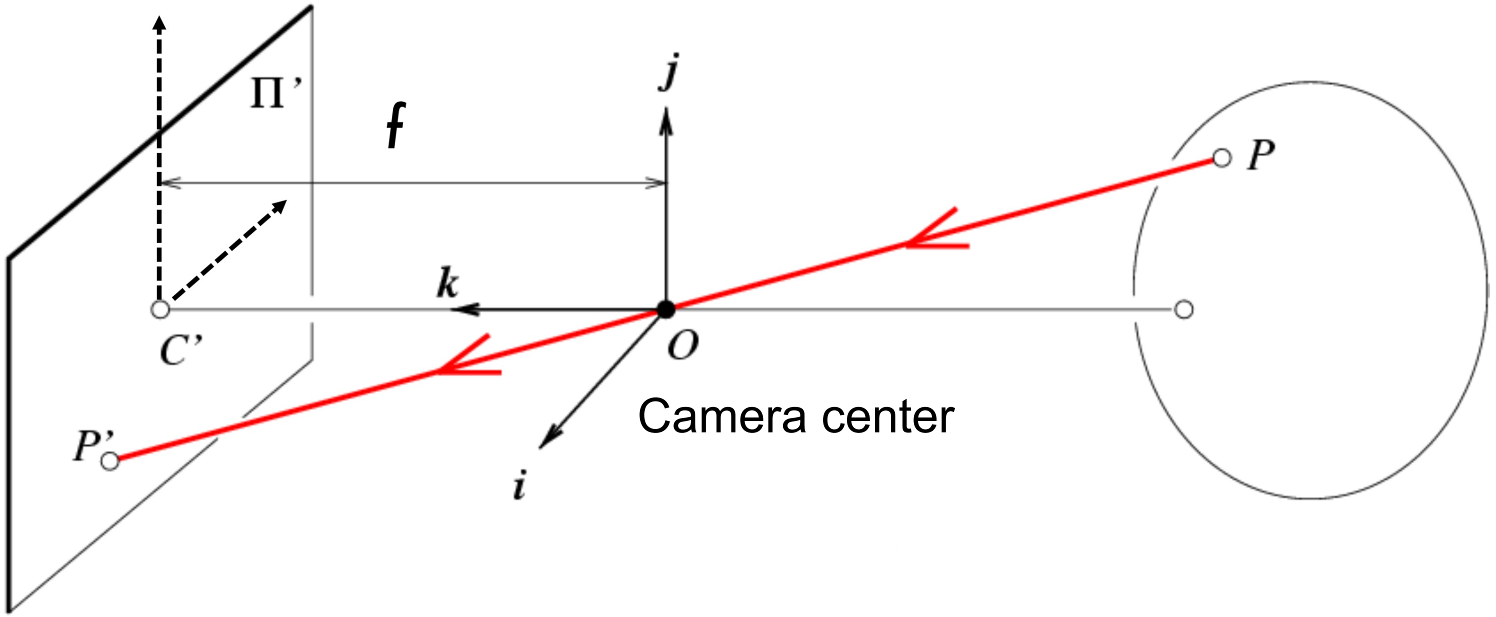
Cannot be implemented in practice
Useful for theoretic analysis

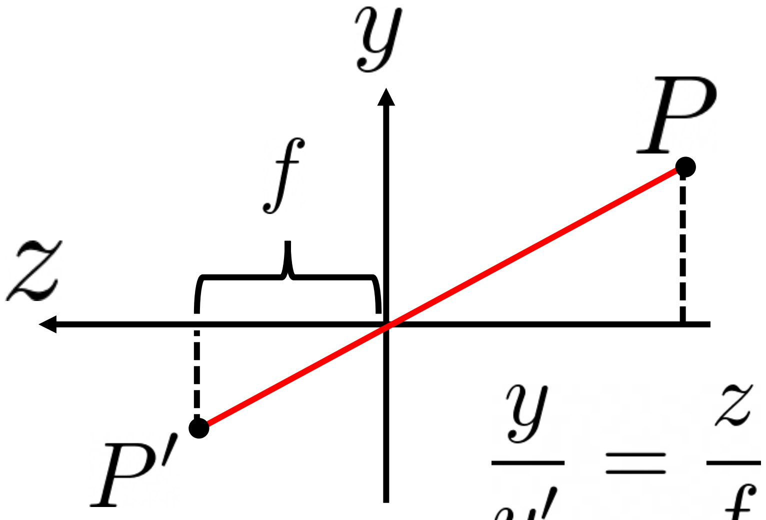
Natural Pinhole Cameras



Object: the sun
Pinhole: gaps between the leaves
Image plane: the ground

Central Projection in Camera Coordinates





$$\frac{y}{y'} = \frac{z}{f}$$

Camera coordinates

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{\text{Nonlinear}} P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

Homogeneous Coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Conversion

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Central Projection with Homogeneous Coordinates

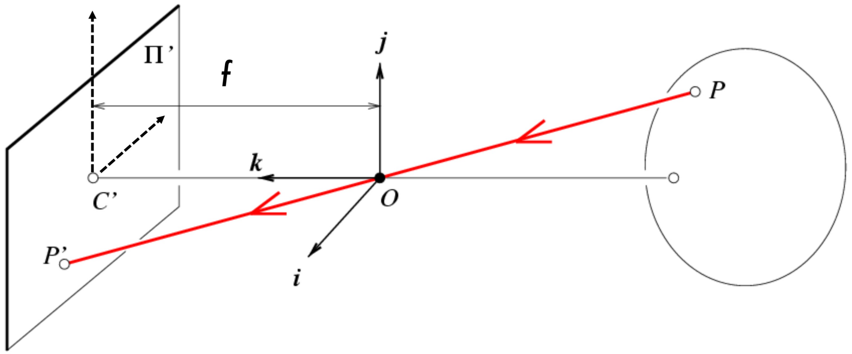
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} f\,\frac{x}{z} \\ f\,\frac{y}{z} \end{bmatrix}$$

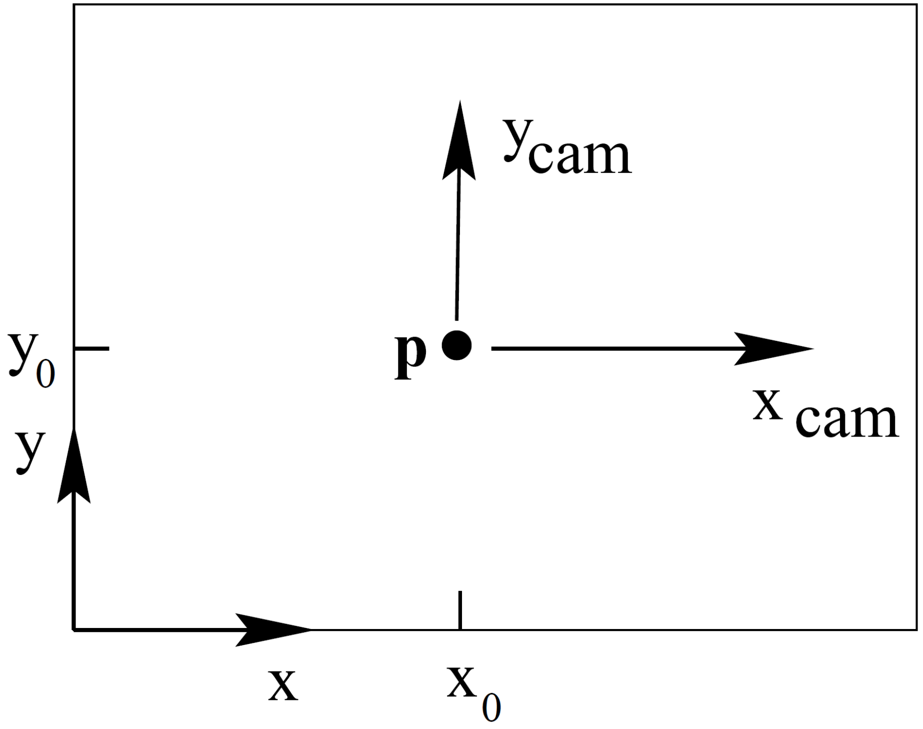
Central projection

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3x4 matrix

Principal Point Offset





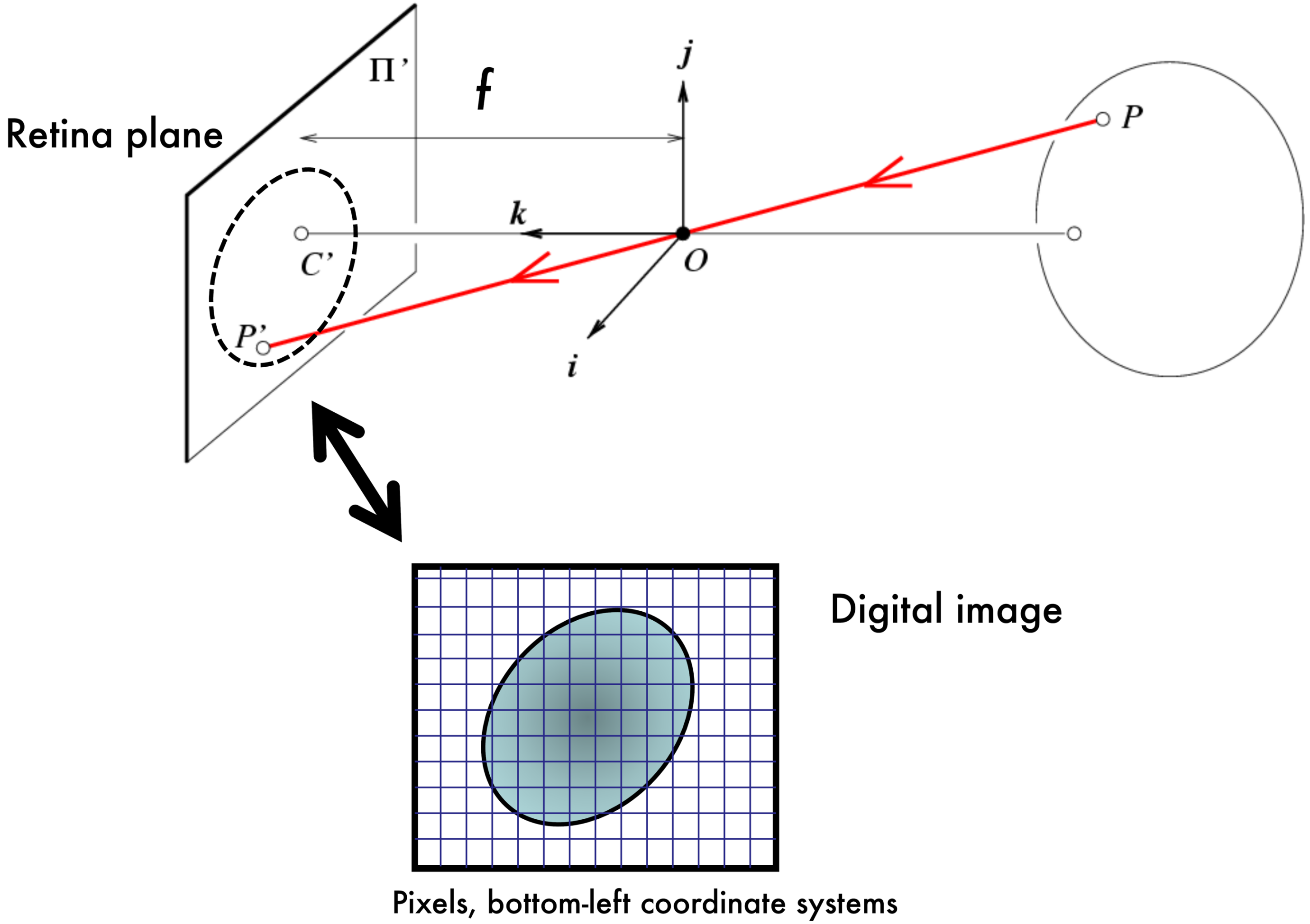
Principle point: projection of the camera center

Principal point $\mathbf{p} = (p_x, p_y)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} f\frac{x}{z} + p_x \\ f\frac{y}{z} + p_y \end{bmatrix}$$

$$\begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

From Metric to Pixels



From Metric to Pixels

Metric space, i.e., meters

$$\begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix}$$

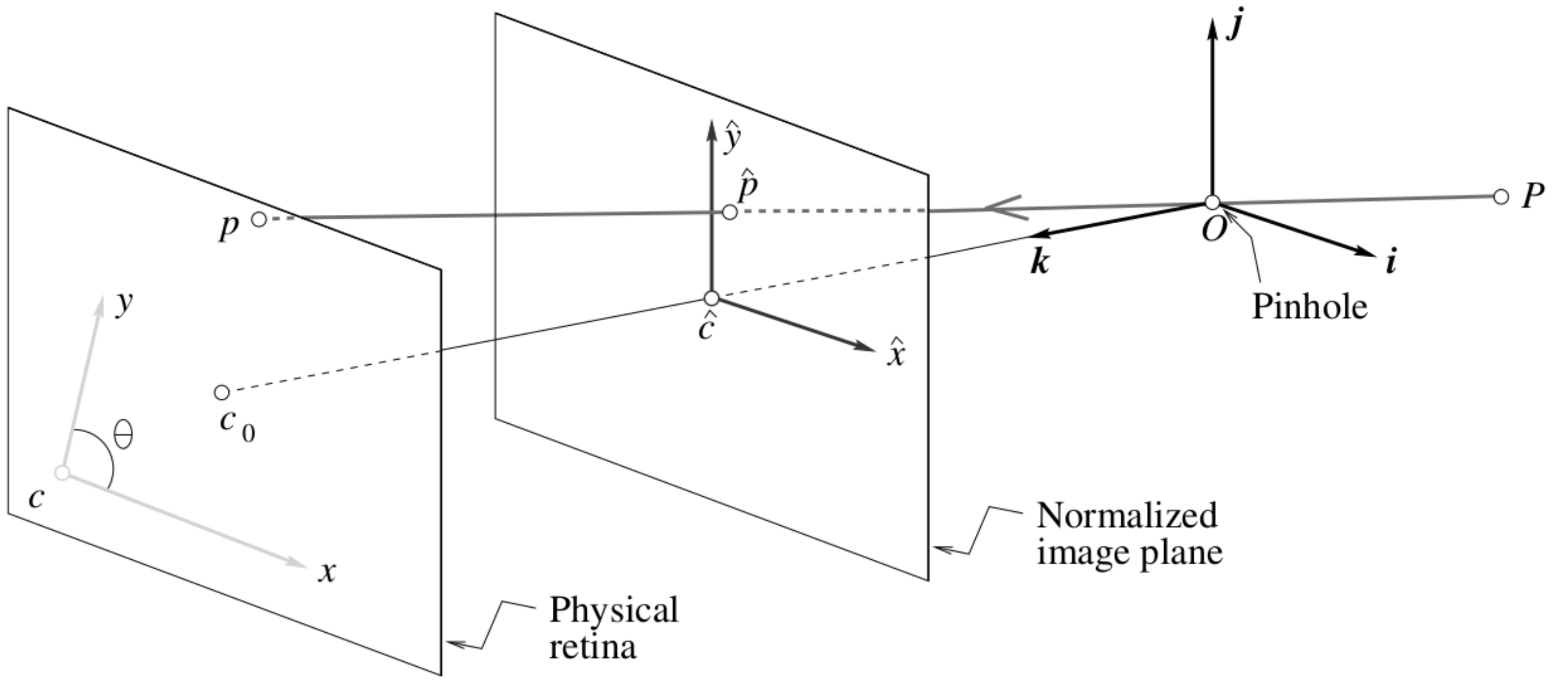
Pixel space

$$\begin{bmatrix} \alpha_x & & x_0 & 0 \\ & \alpha_y & y_0 & 0 \\ & & 1 & 0 \end{bmatrix}$$

$$\alpha_x = f m_x$$
$$\alpha_y = f m_y$$
$$x_0 = p_x m_x$$
$$y_0 = p_y m_y$$

m_x, m_y Number of pixel per unit distance

Axis Skew



The skew parameter will be zero for most normal cameras.

$$\begin{bmatrix} \alpha_x & & x_0 & 0 \\ & \alpha_y & y_0 & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \alpha_x \frac{x}{z} + x_0 \\ \alpha_y \frac{y}{z} + y_0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_x & -\alpha_x \cot(\theta) & x_0 & 0 \\ & \frac{\alpha_y}{\sin(\theta)} & y_0 & 0 \\ & & 1 & 0 \end{bmatrix}$$

https://blog.immenselyhappy.com/post/camera-axis-skew/

Camera Intrinsics

$$\begin{bmatrix} \alpha_x & -\alpha_x \cot(\theta) & x_0 & 0 \\ & \frac{\alpha_y}{\sin(\theta)} & y_0 & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera intrinsics

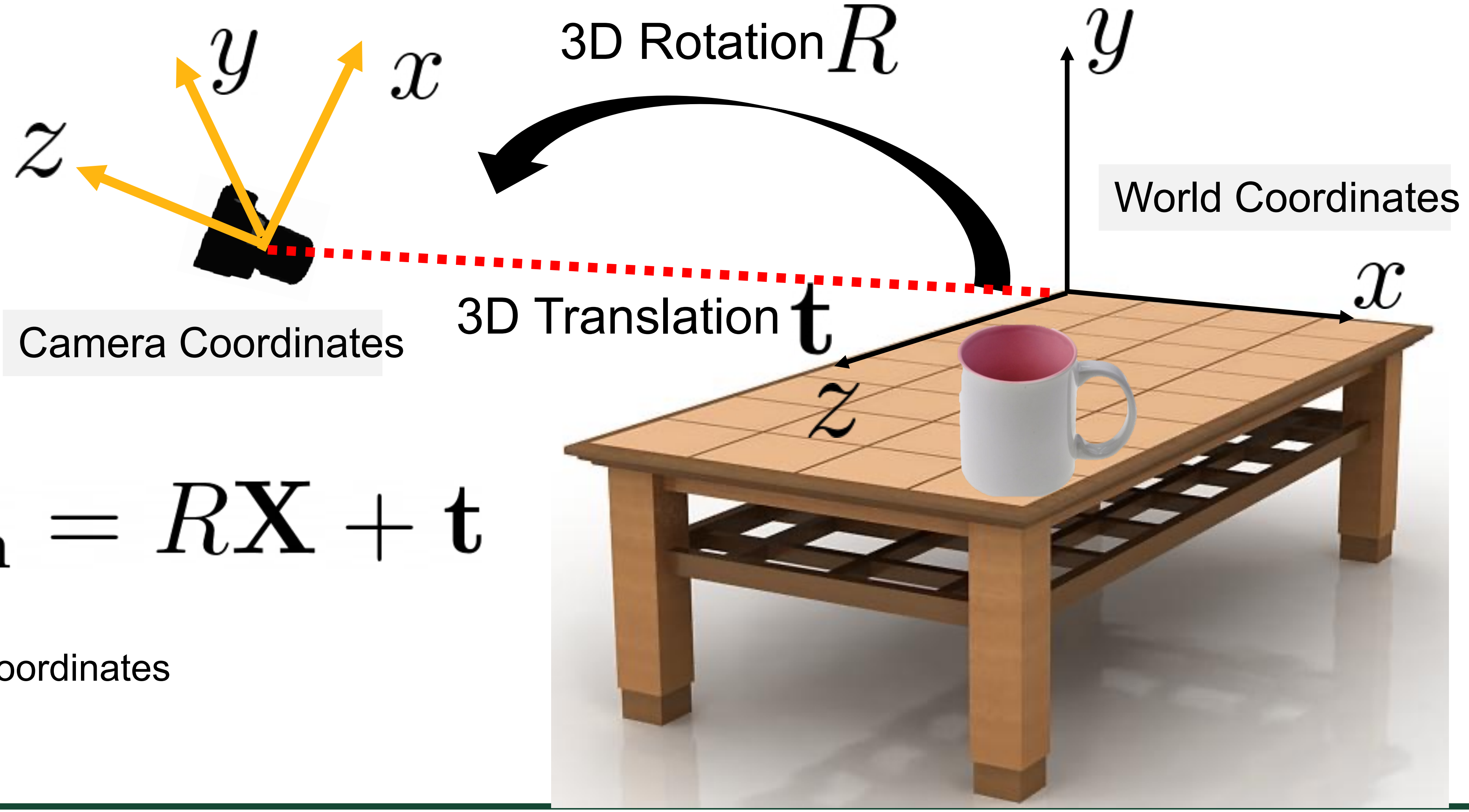
$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\mathrm{cam}}$$

3x1 3x3 3x4 4x1

Homogeneous coordinates

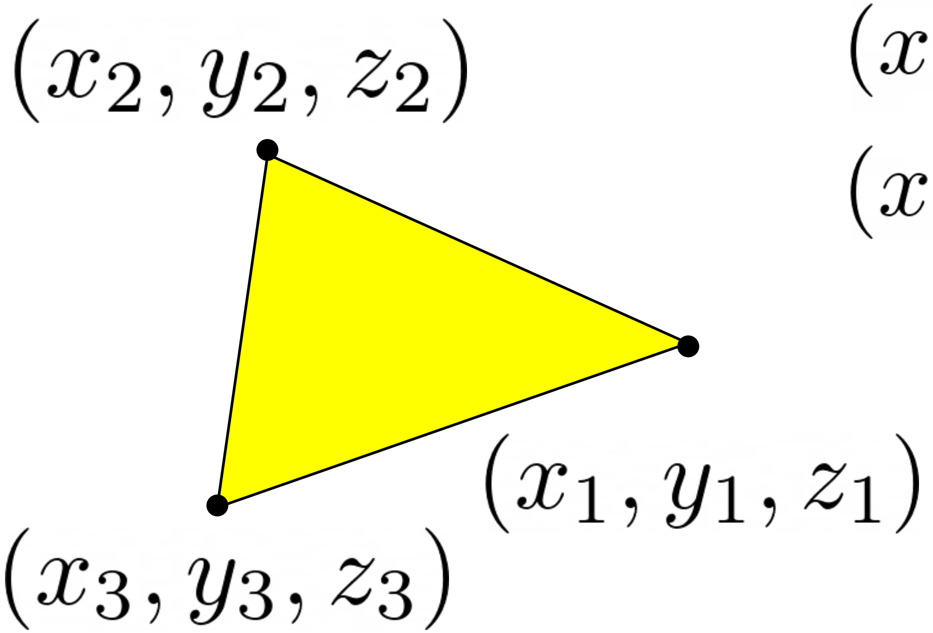
Camera Extrinsics: Camera Rotation and Translation



$$\mathbf{X}_{\mathrm{cam}} = R\mathbf{X} + \mathbf{t}$$

Euclidean coordinates

3D Translation



$$(x_1, y_1, z_1) \mapsto (x_1 + x_t, y_1 + y_t, z_1 + z_t)$$
$$(x_2, y_2, z_2) \mapsto (x_2 + x_t, y_2 + y_t, z_2 + z_t)$$
$$(x_3, y_3, z_3) \mapsto (x_3 + x_t, y_3 + y_t, z_3 + z_t)$$

$$\mathbf{v_1} \mapsto \mathbf{v_1} + \mathbf{t}$$
$$\mathbf{v_2} \mapsto \mathbf{v_2} + \mathbf{t}$$
$$\mathbf{v_3} \mapsto \mathbf{v_3} + \mathbf{t}$$

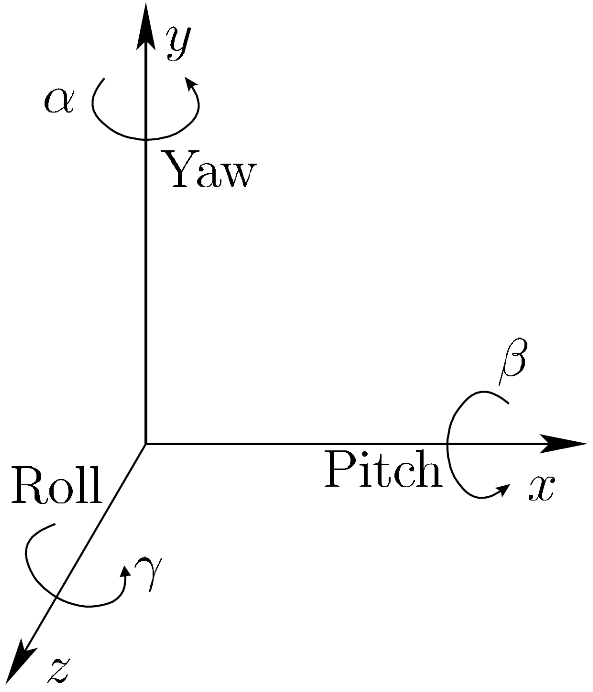
3D Translation $\mathbf{t} = (x_t, y_t, z_t)$

3D Rotation

The yaw, pitch, and roll rotations can be combined sequentially to attain any possible 3D rotation.

$$R(\alpha, \beta, \gamma) = R_y(\alpha) R_x(\beta) R_z(\gamma)$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_x(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \quad R_y(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$



Camera Projection Matrix $P = K[R|\mathbf{t}]$

Homogeneous coordinates

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\text{cam}}$$
$$= K[R|\mathbf{t}]\mathbf{X}$$

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

3x1 3x3 3x4 4x1

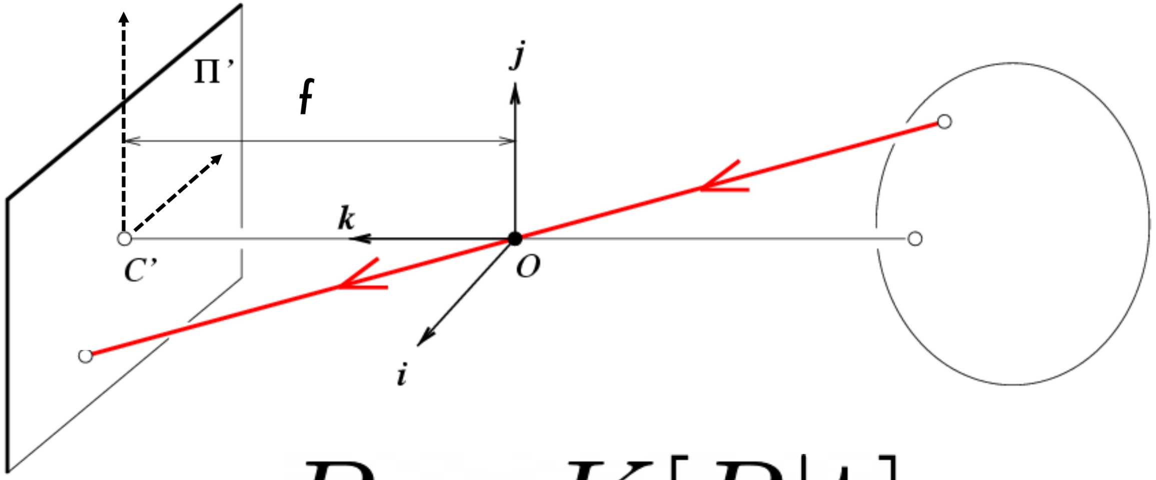
Image coordinates

Camera intrinsics

Camera extrinsics:
rotation and translation

World coordinates

Back-projection in World Coordinates



$$P = K[R|\mathbf{t}]$$

$$\mathbf{x} = P\mathbf{X}$$

- A pixel on the image backprojects to a ray in 3D

- The camera center O is on the ray
- $P^+\mathbf{x}$ is on the ray

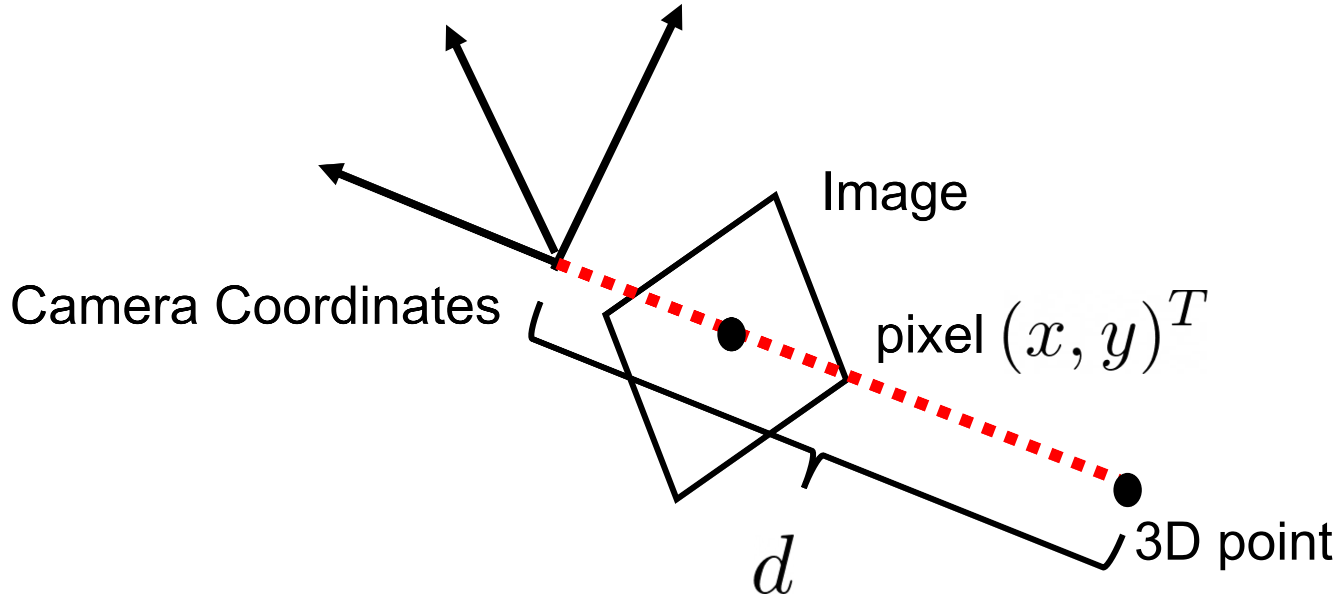
$$P^+ = P^T(PP^T)^{-1}$$

Pseudo-inverse

The ray can be written as

$$P^+\mathbf{x} + \lambda O$$

Back-projection in Camera Coordinates



$$P = K[I|\mathbf{0}]$$

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\mathrm{cam}}$$

$$K^{-1}\mathbf{x}$$

3D point with depth d : $dK^{-1}\mathbf{x}$

3D camera coordinates $\begin{bmatrix} d\frac{x-p_x}{f_x} \\ d\frac{y-p_y}{f_y} \\ d \end{bmatrix}$

Summary: Camera Models

Camera projection matrix: intrinsics and extrinsics

$$P = K[R|\mathbf{t}]$$

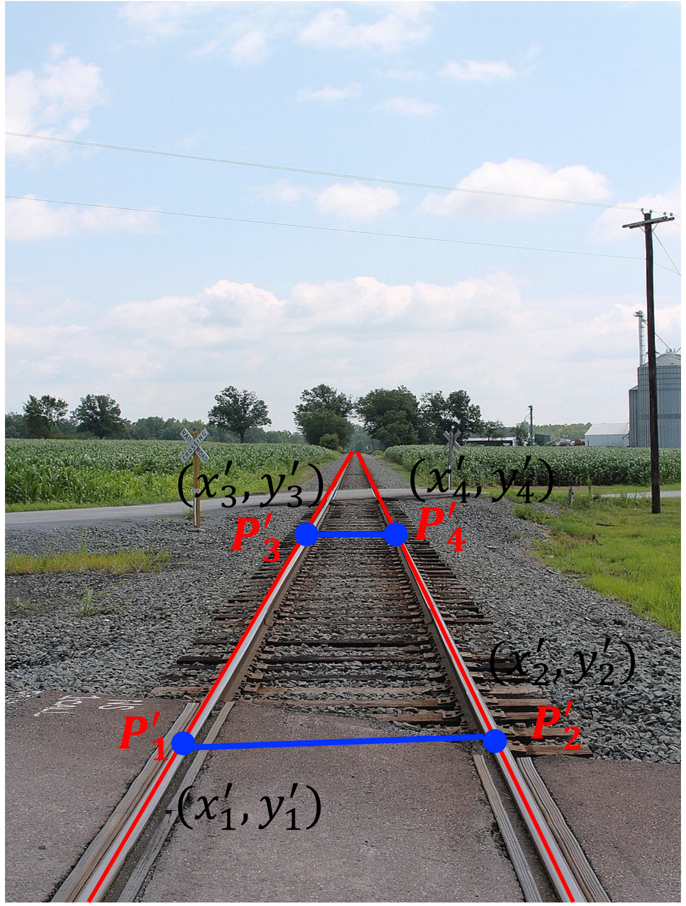
3x3

3x4

Camera intrinsics

Camera extrinsics:
rotation and translation

Interpreting Perceived Images



The lengths of two lines P_1P_2 and P_3P_4 in 3D space are equal

$$\overset{\text{3D}}{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \overset{\text{2D}}{P'} = \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad \begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

Why is $P'_3P'_4$ shorter than $P'_1P'_2$ in the 2D image?

- For the two 3D points P_1 and P_3, let's assume we have $x_1 = x_3, y_1 = y_3$, and $z_1 < z_3$ in the 3D coordinate system
- After 3D-to-2D projection, we have $x'_1 > x'_3$ and $y'_1 > y'_3$
- Larger depth and shorter length due to the projection

Further Reading

Stanford CS231A: Computer Vision, From 3D Reconstruction to Recognition, Course Notes 1: Camera Models

Multiview Geometry in Computer Vision, Richard Hartley and Andrew Zisserman, Chapter 6, Camera Models

Computer Vision: Algorithms and Applications. Richard Szeliski, Chapter 2.1.4, 3D to 2D projections